Optimal monetary policy in developing countries: the role of informality

Monica A. Gomez Ospina*

September 2021

Abstract

In this paper, I analyze optimal monetary policy in developing countries whose labor markets are characterized by the presence of a large informal sector. I develop a closed economy model with nominal price and wage rigidities, search and matching frictions, and a dual labor market: a formal one characterized by matching frictions and nominal wage rigidities, and an informal one where wages are fully flexible. Under this framework, a trade-off between price and wage inflation emerges. I find that informality increases the response of price and wage inflation to aggregate productivity shocks. As a result, the presence of an informal sector increases the inefficient fluctuations of labor market variables, such as unemployment, labor market tightness, and formal hiring rate. I find that optimal policy with informality features significant deviations from price stability in response to aggregate productivity shocks.

JEL classification: E26, E52, E12, E61.

Keywords: Informality, Monetary policy, Nominal wage and price rigidities, Inflation targeting.

1 Introduction

In developing and emerging countries, the informal sector often accounts for a substantial fraction of the urban labor force. According to the International Labor Office (2018), informal

*PhD Candidate at Paris School of Economics, EHESS. Email: monica.gomezos@psemail.eu

employment accounts for more than half of non-agricultural employment in most developing countries: approximately 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States and 50 percent in Latin America. Most of the workers in this sector are self-employed. Their income comes from operating small, unincorporated enterprises[1] that are hidden from regulatory and monetary authorities, and are hardly registered by official statistics. While offering the advantage of employment flexibility in some economies, a large informal sector is associated with low productivity, reduced tax revenues, poverty, and income inequality (World Bank, 2019).

The implications of informality have drawn considerable attention in the literature. Most of the research on this topic studies how informal jobs in the labor market are generated and analyzes the effect of fiscal and labor market policies on informal economic activity. The existing literature focuses mainly on the real economy, and not many papers have been devoted to monetary policy analysis in the presence of a large informal sector. The main objective of this paper is to contribute to this literature by studying the design of optimal monetary policy in economies with informality.

I develop a closed economy model with dual labor markets, formal and informal, that integrates labor market search into a New Keynesian model with nominal price and wage rigidities. Following Thomas (2008) and Gertler and Trigari (2009), I introduce staggered nominal wage bargaining under which firms and workers in the formal sector bargain over wages in a setting with search and matching frictions[2]. Motivated by the fact that the informal sector is mainly characterized by self-employed workers, I assume that wages in the informal sector are flexible.[3]

I obtain the approximated quadratic welfare loss function and then characterize optimal monetary policy under commitment. I find that welfare decreases as inflation and wage volatility increase. Inflation causes inefficient dispersion of prices across retail firms, and similarly, wage inflation generates an inefficient dispersion of wages across formal firms. Welfare also decreases with output and labor market tightness volatility, because the composition of total production between formal and informal goods is distorted when formal labor market tightness differs from its efficient value. Finally, the inefficient fluctuations in employment are an

[1]These include activities such as trading on the streets or in markets; sales of cooked food from kiosks; the transport of people or goods by pedal-power or motorbikes; repairing clothes, shoes, or motor scooters; dwelling construction or adding extensions to them; scavenging for reusable waste; or providing a range of personal services such as hairdressing, fortune-telling, shoe cleaning, street theater, house cleaning, and the like (Blades et., al. 2011).

[2]Under this setting, formal wages will affect employment at an extensive margin. They influence the rate at which firms in the formal sector add new workers to their respective labor forces. As emphasized by Hall (2005), in this kind of setting the Barro's critique does not apply (Gertler and Trigari, 2009).

[3]For a discussion of the reasons why the informal sector should have less frictions, see Zenou (2008)

additional source of welfare losses. To my knowledge, this is the first paper to introduce both nominal price and wage rigidities in a model with informality and to characterize optimal monetary policy in these types of economies.

I show that for the case in which only price rigidities are present, wages in the formal sector are Nash bargained every period, and the steady state is efficient, a zero inflation policy is optimal. I also show that when the negotiation of formal wages is staggered, a trade-off between price inflation and unemployment stabilization emerges. In the presence of price and formal wage rigidities, complete price-level stabilization is no longer optimal. As a consequence, the central bank should consider both price and formal wage stability, since fluctuations in price and wage inflation generate inefficient fluctuations in the allocation of resources in the economy.

To better understand the implications of informality for optimal monetary policy, I compare the predictions of the model against a case in which there is no informal sector in the economy. I find that the contribution of wage inflation volatility to welfare loss, relative to the contribution of price inflation volatility, is lower for the case with informality. This result is explained by the fact that in the presence of an informal sector, the proportion of firms in the economy facing wage rigidities is lower. Therefore, in the presence of an informal sector, the optimal policy will result in a lower price inflation volatility for a given level of wage inflation volatility.

Additionally, I find that, in the presence of an informal sector, the inefficient fluctuations in labor market variables such as employment, labor market tightness, and formal hiring rate are higher. This result is because, in response to an aggregate productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. This wage rigidity generates a gap between the actual and the natural formal wage (the target wage) that translates into wage dispersion and inefficient job creation in the formal sector. In the presence of informality, the response of the target wage to productivity shocks is higher. The target wage in the formal sector depends on the informal wage (the outside option) and formal labor market tightness. After a negative productivity shock, the decrease in both variables is higher than in the case without informality: on the one hand, the outside option decreases with an adverse productivity shock; and on the other hand, the informal sector works as a buffer that absorbs workers in bad times, and vice-versa. Consequently, after a negative productivity shock, the increase in unemployment is lower in the presence of an informal sector. Hence, the probability of filling a formal vacancy is also lower, pinning down the formal firm's surplus and their incentive to hire.

As a result, in response to aggregate productivity shocks, the central bank should use price inflation to avoid excessive wage inflation volatility that causes excessive unemployment volatility and excessive dispersion in the formal hiring rate. By controlling the inflation rate, the central bank can affect the real value of nominal wages and then bring real formal wages closer to their flexible-wage levels. The presence of an informal sector requires a higher adjustment of price inflation to reduce this gap.

In summary, the existence of a large informal sector has two opposite implications for optimal monetary policy. On the one hand, given that in the presence of informality the proportion of firms in the economy facing wage rigidities is lower, the optimal policy results in a lower price inflation volatility for a given level of wage inflation volatility. On the other hand, in the presence of a large informal sector, wage inflation and unemployment volatility are higher. As a result, the central bank should use price inflation to avoid excessive unemployment volatility and excessive dispersion in the formal hiring rate. The aggregate effect on price inflation volatility would depend on which effect dominates. For a standard calibration of the model for an economy with a large informal sector, I find that under the optimal monetary policy, the volatility of inflation is relatively higher in the presence of informality.

Finally, to illustrate the implications of the trade-off faced by the central bank, I analyze the behavior of a decentralized economy when the monetary authority implements a policy of zero price inflation. I find that the welfare loss under a zero price inflation policy is approximately 1.26 times as large as under the optimal policy. For the case without informality, the welfare loss under a zero price inflation policy is approximately 0.015 times larger than the regime under the optimal policy. These results show that a policy designed to minimize price inflation volatility can generate significant welfare losses in the presence of nominal wage rigidities and informality, which might be the case for most emerging countries.

1.1 Related literature

Early modeling in the area of informal economics started from the classic Harris-Todaro (1970) framework. In these models, informality is captured by building a model of two distinct markets that are segmented and in which two different wage equilibria prevail (wage duality), where wages in the formal sector can turn out to be higher than the market-clearing wages. Brueckner and Zenou (1999) add a land market to the standard Harris-Todaro framework where wages are endogenously fixed. The idea of identifying the informal labor market with the disadvantaged sector of a market segmented by rigidities in the formal sector dates back to Lewis (1954). Most recent literature has developed more sophisticated models to represent formal, informal, and integrated labor markets (see Boeri and Garibaldi 2005, Fugazza and Jacques 2003, and Badaoui et al. 2006). In these models, trading frictions in the formal and informal sectors are important, and it is possible to determine rules governing the flows between the two sectors, as well as to and from the pool of the unemployed. Most of these models incorporate the search and matching model of Mortensen-Pissarides into the Harris and Todaro model.

The previous literature focuses on the intersectoral margin for workers and firms. Other papers like Albrecht et al. (2008), Zenou (2008), and Satchi and Temple (2009) focus on the intersectoral margin for workers. Albrecht et al. (2008) develop a search and matching model with endogenous job destruction. Workers have the same productivity in the informal sector (unregulated self-employment), while they have different productivity if they decide to enter the formal sector. In this way, the relative productivity in the two sectors is an important factor in the workers choice. In this model, unemployment is the residual state in the sense that workers whose employments ends, in either the informal or the informal sector, flow back into unemployment.

In general, these papers aim to study how informal jobs in the labor market are created, and the effect of fiscal policy and labor market institutions (such as employment protection legislation, tax wedge, unemployment benefits, unemployment benefit duration, and union density) on informal economic activity. These studies focus on the real economy and do not analyze the interaction between the informal sector and monetary policy. Surprisingly, few papers have been devoted to monetary policy analysis when the economy displays a large informal sector. An exception is Castillo and Montoro (2010), Batini et al. (2011), and Alberola and Urrutia (2020).

Castillo and Montoro (2010) is the first paper that analyses the effect of informal labor markets on monetary policy. They extend Blanchard and Gali (2010) by modeling a dual labor market economy with formal and informal labor contracts within a New Keynesian model with labor market frictions. In this framework, informality is a result of hiring costs, which are a function of the ratio of vacancies to unemployment. They find that informal workers act as a buffer on employment that allows firms to increase output without generating pressure on wages. Batini et al. (2011) study how informality affects the conduct of monetary policy. They develop a two-sector, formal and informal, New Keynesian model. The informal sector is more labor-intensive, can avoid taxation, has a classical labor market, faces high credit constraints in financing investment, and is less visible in terms of observed output. They find that the importance of commitment increases in economies characterized by a large informal sector and that optimal simple rules that respond only to observed aggregate inflation and formal output can be significantly worse in welfare terms than their optimal counterpart. In the same line, Arberola and Urrutia (2020) analyze the effect of informality on monetary policy. They develop a general equilibrium closed economy model with labor and financial frictions and nominal price rigidities. They find that informality has a buffering effect on the propagation of demand and supply shocks to prices. As a result, informality dampens the impact of demand and financial shocks on wages and inflation but amplifies the impact of technology shocks. Informality also increases the sacrifice ratio of monetary policy.

In contrast to Castillo and Montoro (2010), Alberola and Urrutia (2020), and Batini et al. (2011), I consider both price and wage rigidities and characterize optimal monetary policy under commitment. Under this framework, it is possible to analyze optimal monetary policy with informality in a scenario where there is a trade-off between inflation and unemployment.

1.2 Outline

The rest of the paper is organized as follows: Section 2 presents the model. In section 3, for comparative purposes, I consider both the equilibrium of the model with flexible price and wages, and the Social Planner Solution. In section 4, I derive a log-linear approximation of the rational expectations equilibrium around the efficient steady state under staggered wage bargaining in the formal sector. Section 5 analyzes the optimal monetary policy under commitment and the role of informality on the optimal monetary policy design. Section 6 concludes.

2 Model

The analysis builds on a New Keynesian framework with dual labor markets. The model consists of *households* whose utility depends on the consumption of market goods and whose members are either employed in the formal or informal sector or are unemployed. *Wholesale formal firms* employ formal labor to produce a wholesale formal good that is sold in a competitive market. The labor market in this sector is characterized by search and matching frictions and nominal wage rigidities. *Wholesale informal firms* employ informal labor to produce a wholesale informal good sold in a competitive market. The labor market in this sector is fully flexible (prices and wages are flexible, and there are no search and matching frictions). *Retail firms* aggregate the two wholesale goods and transform them into differentiated final goods that are sold to households in an environment of monopolistic competition.

2.1 Wholesale firms

I assume two types of firms in the wholesale sector: formal and informal. *Formal wholesale firms* produce a homogeneous formal intermediate good that is sold to retailers at a competitive price p_t^f. The labor market in this sector is characterized by the presence of search and matching frictions and staggered wage bargaining. On the other side, *informal wholesale firms* produce a homogeneous informal intermediate good that is sold to retailers at a competitive price $p_t^\imath$. The labor market in this sector is not subject to search and matching frictions and wages are flexible.

2.1.1 Informal wholesale firms

Every period, each worker in the informal sector produces $y_{it}^\imath$ units of output under a production technology linear in labor $l_t^\imath$.

The aggregate output of the informal sector is given by:

$$y_t^\imath = z_t z^\imath l_t^\imath, \tag{1}$$

where z_t is an aggregate productivity shock, and $z^\imath$ is a parameter denoting the productivity associated with workers in the informal sector. $ln(z_t)$ follows a first-order auto-regressive process, $ln\,(z_t) = \rho_z ln(z_{t-1}) + \varepsilon_t^z$, where ε_t^z is an independent and identically distributed shock.

Workers in this sector are self-employed, and wages equal the marginal productivity of labor:

$$w_t^\imath = p_t^\imath z_t z^\imath. \tag{2}$$

2.1.2 Formal wholesale firms

The matching function

Formal wholesale firms produce a homogeneous formal intermediate good y_t^f. In this sector, the number of hires is determined by a search and matching process. Each period, the number of successful matches between firms that post vacancies v_t and unemployed workers looking for a job in the formal sector l_t^u is determined by the matching function:

$$m(v_t,\, l_t^u) = \mathbb{N}\,(l_t^u)^\mu\,(v_t)^{1-\mu}\,, \tag{3}$$

where $\mathbb{N}$ is a scale parameter that reflects the efficiency of the matching process, and $(1-\mu) \in (0,1)$ measures the elasticity of the matching function with respect to vacancies.

The probability of filling a vacancy, $q(\theta_t)$, is equal to:

$$q(\theta_t) = \frac{m(v_t,\, l_t^u)}{v_t} = \mathbb{N}(\theta_t)^{-\mu}\,, \tag{4}$$

where $\theta_t = \frac{v_t}{l_t^u}$ is the labor market tightness in the formal sector.

Similarly, the probability that an unemployed worker finds a job in the formal sector, $p(\theta_t)$, is equal to:

$$p(\theta_t) = \frac{m(v_t,\, l_t^u)}{l_t^u} = \mathbb{N}(\theta_t)^{1-\mu}\,, \tag{5}$$

Equation (5) implies that an increase in the number of vacancies relative to the number of unemployed individuals who search for a job in the formal sector increases the probability for an unemployed person of finding a job in this sector. In contrast, an increase in θ_t decreases the probability of filling a vacancy. Both firms and workers take $q(\theta_t)$ and $p(\theta_t)$ as given.

Assuming that firms in this sector are sufficiently large, $q(\theta_t)$ represents the fraction of vacancies filled in period t. New hires do not become productive until the next period, given the time involved in recruiting and training these new workers.

Therefore, the aggregate of employed workers in the formal sector at time $t+1$ can be represented as follows:

$$l_{t+1}^f = (1-\rho)\, l_t^f + q(\theta_t)\, v_t, \tag{6}$$

where ρ is the exogenous destruction rate of formal employment.

Formal production

In the formal sector, firms are indexed by i. Each firm employs l_{it}^f workers in period t and posts v_{it} vacancies to attract new workers for the next period of operation. The total number of vacancies and employed workers in the formal sector are $v_t = \int_i v_{it} di$ and $l_{it}^f = \int_i l_{it}^f di$, respectively. If the search process is successful, the firm in the formal sector operates with the following technology:

$$y_{it}^f = z_t z^f l_{it}^f,$$

where z^f is a parameter that represents labor productivity specific to the formal sector, with $z^f > z^\imath$. y^f_{it} is sold to retailers at a price p^f_t.

The hiring rate $\mathcal{F}_{it}$, is defined as the ratio between the number of vacancies v_{it}, and the number of hired workers l^f_{it}:

$$\mathcal{F}_{it} = \frac{v_{it}}{l^f_{it}}.$$

Because of the staggered wage bargaining process, there will be wage dispersion across firms. As in Thomas (2008) and Gertler and Trigari (2009), I assume convexity in the vacancy-posting cost to ensure an equilibrium where all formal firms post vacancies in the presence of wage dispersion[4]. This cost is measured in terms of utility for the firm's management, and is given by:

$$\frac{\kappa}{2}\mathcal{F}^2_{it}l^f_{it}.$$

Conditional on the current wage and employment, the present value of the flow of benefits Q^o_{it} for each firm in the formal sector, can be expressed as:

$$Q^o_{it} = \underbrace{max}_{v_{it}}\left\{p^f_t y^f_{it} - w^f_{it}l^f_{it} - \frac{\kappa}{2}\mathcal{F}^2_{it}\frac{l^f_{it}}{u'(c_t)} + E_t\Gamma_{t,t+1}Q^o_{it+1}\right\},$$

subject to the law of motion of employment in firm i:

$$l^f_{it+1} = (1-\rho)\,l^f_{it} + q\,(\theta_t)\,v_{it}. \tag{7}$$

$\Gamma_{t,t+s} = \beta^s\frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor between periods t and $t+s$. Firms choose the hiring rate by setting the number of vacancies in period t. They maximize the present value of the flow of benefits, taken as given the probability of filling a vacancy and the current path of expected wages. In case firm i can renegotiate wages, it bargains with its workforce over a new contract. Otherwise, the firm sets the wage at the level of the previous period. The first-order condition with respect to vacancies is given by:

$$\frac{\kappa\mathcal{F}_{it}}{u'(c_t)} = q\,(\theta_t)\,E_t\Gamma_{t,t+1}\frac{\partial Q^o_{it+1}}{\partial l^f_{it+1}}. \tag{8}$$

The value of the marginal worker for the firm is given by:

$$J^f_{it} = \frac{\partial Q^o_{it}}{\partial l^f_{it}} = p^f_t mpl^f_t - w^f_{it} + \frac{\kappa\mathcal{F}^2_{it}}{2u'(c_t)} + (1-\rho)E_t\Gamma_{t,t+1}\frac{\partial Q^o_{it+1}}{\partial l^f_{it+1}}. \tag{9}$$

[4] Wage dispersion creates a dispersion in the marginal benefit of posting vacancies. When the cost of a vacancy is linear, then the marginal cost of posting vacancies would be the same for all firms, and only the firm with the lower wage would post vacancies.

Therefore, the value for a formal firm of having an occupied job at time t is equal to the marginal product of a worker mpl_t^f, minus the real wage w_{it}^f, plus the saving on adjustment costs, plus the discounted value of having a match in the following period. Combining equations (8) and (9) yields the following condition for the hiring rate:

$$\frac{\kappa \mathcal{F}_{it}}{q(\theta_t)} = \beta E_t \left[u'(c_{t+1}) \left(p_{t+1}^f mpl_{t+1}^f - w_{it+1}^f + \frac{\kappa \mathcal{F}_{it+1}^2}{2u'(c_{t+1})} \right) + (1-\rho) \frac{\kappa \mathcal{F}_{it+1}}{q(\theta_{t+1})} \right]. \tag{10}$$

Equation (10) equates the cost of hiring a worker, discounted by the probability of filling a vacancy, to the expected value of a match. The hiring rate depends on the discounted streams of benefits from having a filled job, plus the savings on adjustment costs. Note that the wage w_{it}^f set by firm i, is the only firm-specific variable that affects the hiring rate $\mathcal{F}_{it}$. Consequently, all firms with the same wage w_t^f are going to choose the same hiring rate, independent of their respective employment size.

The dividends that the household receives from formal firms are equal to:

$$div_{it}^{yf} = p_t^f y_{it}^f - w_{it}^f l_{it}^f.$$

Given the constant returns to scale in production, it is possible to express aggregate output of the formal intermediate good as follows:

$$y_t^f = z^f z_t \int_0^1 l_{it}^f di = z^f z_t l_t^f.$$

2.2 Retailers

In the retail sector, there is a continuum of monopolistic competitive retailers indexed by j on the unit interval. Let y_j be the quantity of output sold by retailer j. Retail firms use an aggregate of intermediate goods to produce a final differentiated good. The aggregate intermediate good, is a composite of formal and informal goods, according to the Constant Elasticity of Substitution (CES) aggregator:

$$y_{jt} = \left[\left(y_t^f \right)^{\frac{\gamma-1}{\gamma}} + \left(y_t^{\imath} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{11}$$

where γ is the elasticity of substitution between formal and informal produced goods.

To determine the demand for y_t^f and $y_t^{\imath}$, retailers solve the following minimization cost problem:

$$min\ p_t^{\imath} y_t^{\imath} + p_t^f y_t^f,$$

subject to (11).

The First Order Conditions (F.O.C) imply:

$$y_{it}^f = \left(\frac{mc_t}{p_t^f}\right)^{\gamma} y_{jt}, \qquad y_{it}^{\imath} = \left(\frac{mc_t}{p_t^{\imath}}\right)^{\gamma} y_{jt},$$

where

$$mc_t = \left[\left(p^f\right)^{1-\gamma} + \left(p_t^{\imath}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

represents the real marginal cost of producing an additional unit of y_t.

The total production of final goods is equal to the following composite of individual retail goods:

$$y_t = \left[\int_o^1 \left(y_{jt}^{\frac{\Theta-1}{\Theta}}\right)\right]^{\frac{\Theta}{\Theta-1}},$$

where Θ is the elasticity of substitution between differentiated goods. In line with Calvo (1983), retail firms can change their prices optimally every period with a probability $(1-\omega^p)$ and set a price P_t^*. With probability ω^p the firm will set the price of the previous period P_{t-1}.

If the firm has the chance to set prices optimally, it will choose the price that maximizes the present discounted value of the firm's benefits, as follows:

$$\underset{P_t^*}{max} \quad E_t \sum_{\ell=0}^{\infty} \Gamma_{t,t+\ell} \left(\omega^p\right)^{\ell} \left[\left(1+\tau^m\right) P_t^* y_{t+\ell/t} - MC_{t+\ell/t} y_{t+\ell/t}\right]$$

subject to the sequence of demand constraints:

$$y_{t+\ell/t} = \left(\frac{P_t^*}{P_{t+\ell}}\right)^{-\Theta} y_{t+\ell}. \tag{12}$$

where $y_{t+\ell/t}$ and $MC_{t+\ell/t}$ denote, respectively, the output and nominal marginal cost in period $t+\ell$ for a firm whose last reset of prices was in period t.

To offset the distortion caused by monopolistic competition in the retail sector and to ensure that the steady-state equilibrium is efficient, I assume that the firm's output is subsidized at the fixed rate $\tau^m = \frac{1}{\Theta}$. The optimal firm's price-setting decision is given by (see Appendix B2 for the derivation):

$$\frac{P_t^*}{P_t} = \frac{E_t \sum_{\ell=0}^{\infty} \beta^\ell \left(\omega^p\right)^\ell \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{-\Theta} mc_{t+\ell}}{E_t \sum_{\ell=0}^{\infty} \beta^\ell \left(\omega^p\right)^\ell \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta}}. \tag{13}$$

2.3 Households

The representative household consists of an extended family that contains a continuum of members. In this household, a fraction $l_t^f = \int_0^1 l_{it}^f di$ of its members are employed in the formal sector, where l_{it}^f represents the number of workers in a firm i. A fraction $l_t^\imath$ is working in the informal sector (self-employed), and the remaining fraction $l_t^u = 1 - l_t^f - l_t^\imath$ is unemployed and searching for a job in the formal sector. Following most of the literature, I assume the existence of a representative infinitely-lived household, where all members pool their income and consumption is equalized across members. The representative household maximizes the following utility function:

$$U_t = \sum_{t=0}^{\infty} \left\{ u\left(c_t\right) - \varphi(l_t^f + l_t^\imath) \right\},$$

where

$$c_t = \left[\int_0^1 \left(c_{jt}\right)^{\frac{\Theta-1}{\Theta}} dj \right]^{\frac{\Theta}{\Theta-1}}$$

is an aggregate of differentiated final goods purchased from the continuum of retail firms, indexed by $j \in [0,1]$. The function $u\left(c_t\right)$ is strictly increasing and strictly concave. Following Ravena and Walsh (2011) and Tomas (2008), I introduce a fixed component of the disutility of working φ[5].

The demand for each differentiated consumption good is determined by the intratemporal optimal choice across goods. It implies:

$$c_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\Theta} c_t, \tag{14}$$

where

$$P_t = \left[\int_0^1 \left(P_{jt}\right)^{\frac{\Theta-1}{\Theta}} dj \right]^{\frac{\Theta}{\Theta-1}} \tag{15}$$

[5]Under this framework, it is possible to assume linearity in work disutility due to the assumption of risk sharing and labor supply changing at the extensive margin (see Rogerson, 1988)

is the Dixit–Stiglitz aggregate price index. The law of motion for the price level is given by:

$$P_t^{1-\Theta} = \omega^p \left(P_{t-1}\right)^{1-\Theta} + \left(1-\omega^p\right)\left(P_t^*\right)^{1-\Theta}.$$

In each period, the household faces the following budget constraint:

$$\int_0^1 w_{it}^f l_{it}^f di + w_t^\imath l_t^\imath + div_t^{yf} + \frac{(1+r_{t-1})}{(1+\pi_t)} b_{t-1} = c_t + b_t, \tag{16}$$

where div_t^{yf} are the dividends from the formal intermediary firms, r_t is the nominal interest rate, b_t are bonds and π_t is the inflation rate. The household chooses c_t and b_t that maximize their expected discounted utility, subject to their budget constraint. The first-order condition for this optimization problem results in the standard Euler equation:

$$u'\left(c_t\right) = \beta E_t u'\left(c_{t+1}\right) \frac{(1+r_t)}{(1+\pi_{t+1})}, \tag{17}$$

where $u'\left(c_t\right)$ is the marginal utility of consumption.

In equilibrium, the total supply of the final good y_t must equal total demand by households $\int_0^1 c_{jt} d_j$. This condition can be written as follows:

$$y_t = \triangle_t c_t, \tag{18}$$

where $\triangle_t = \int_0^1 \left(\frac{p_{jt}}{p_t}\right)^{-\Theta} dj$ is a measure of the price dispersion.

2.3.1 Worker's value functions

The present discounted value for a worker in the formal sector is:

$$\mathbb{Q}_{it}^f = w_{it}^f - \frac{\varphi}{u'(c_t)} + E_t \Gamma_{t,t+1} \Big((1-\rho)\mathbb{Q}_{it+1}^f + \rho \max\left[\mathbb{Q}_{t+1}^{lu}, \mathbb{Q}_{t+1}^\imath\right]\Big). \tag{19}$$

Equation (19) implies that a worker hired in the formal sector receives a real wage $w_{it}^f = \frac{W_{it}^f}{P_t}$ and has a disutility of working equal to $\frac{\varphi}{u'(c_t)}$. In the next period, she will continue working in this sector with probability $(1-\rho)$, in which case she will obtain an expected value of $\mathbb{Q}_{t+1}^f$. The probability that a formal worker loses her job is ρ, in which case she will decide whether to become unemployed or work in the informal sector. This decision will depend on the maximal value between $\mathbb{Q}_{t+1}^{lu}$ and $\mathbb{Q}_{t+1}^\imath$.

Additionally, the present discounted value for a worker in the informal sector is:

$$\mathbb{Q}_{it}^\imath = w_{it}^\imath - \frac{\varphi}{u'(c_t)} + E_t \Gamma_{t,t+1} \max\left[\mathbb{Q}_{t+1}^{lu}, \mathbb{Q}_{t+1}^\imath\right]. \tag{20}$$

In this case, a worker in the informal sector receives a real wage $w_{it}^{\imath} = \frac{W_{it}^{\imath}}{P_t}$ and has a disutility of working equal to $\frac{\varphi}{u'(c_t)}$. To apply for formal jobs, informal workers have to become unemployed. Therefore, in the next period workers in this sector either will become unemployed or will continue working in the informal sector depending on the $\max\left[\mathbb{Q}_{t+1}^{lu}, \mathbb{Q}_{t+1}^{\imath}\right]$.

Finally, the present discounted value for an unemployed worker is equal to:

$$\mathbb{Q}_t^{lu} = E_t \Gamma_{t,t+1} \left(p\left(\theta_t\right) \bar{\mathbb{Q}}_{\mathcal{F},t+1}^{f} + \left(1 - p\left(\theta_t\right)\right) \max\left[\mathbb{Q}_{t+1}^{lu}, \mathbb{Q}_{t+1}^{\imath}\right] \right), \tag{21}$$

where $\bar{\mathbb{Q}}_{\mathcal{F},t}^{f} = \int_0^1 \mathbb{Q}_{it}^{f} di$ is the average value of employment in the formal sector. The probability of finding a job in the formal sector in period t is $p\left(\theta_t\right)$. In this case, they will start working in the next period and obtain an expected value of $\bar{\mathbb{Q}}_{\mathcal{F},t+1}^{f}$. With probability $\left(1 - p\left(\theta_t\right)\right)$ they do not find a job in the formal sector, in which case they either will continue to be unemployed or will go to work in the informal sector, depending on the $\max\left[\mathbb{Q}_{t+1}^{lu}, \mathbb{Q}_{t+1}^{\imath}\right]$. In equilibrium, the present discounted value for an unemployed individual equals the present discounted value of a worker in the informal sector, $\mathbb{Q}_t^{lu} = \mathbb{Q}_t^{\imath}$. Note that in the presence of staggered wage bargaining, the present value of finding a formal job in the next period for a worker who is currently unemployed is $\bar{\mathbb{Q}}_{\mathcal{F},t+1}^{f}$. This is because the unemployed worker does not know in advance which firm would be paying higher wages in the next period. The unemployed agent can only choose randomly among formal firms posting vacancies.

The surplus derived by the worker at the firm paying a real wage w_{it}^{f}, is denoted as $\mathbb{H}_{it}^{f}$, and $\mathbb{H}_{\mathcal{F},t}^{f}$ denotes the average formal worker's surplus conditional on being a new hire, which are defined as follows:

$$\mathbb{H}_{it}^{f} = \mathbb{Q}_{it}^{f} - \mathbb{Q}_t^{lu}$$

$$\mathbb{H}_{\mathcal{F},t}^{f} = \bar{\mathbb{Q}}_{\mathcal{F},t}^{f} - \mathbb{Q}_t^{lu}.$$

Worker surplus in the formal sector can be expressed as:

$$\mathbb{H}_{it}^{f} = w_{it}^{f} - \frac{\varphi}{u'\left(c_t\right)} + E_t \Gamma_{t,t+1} \left[(1-\rho) \mathbb{H}_{it+1}^{f} - p\left(\theta_t\right) \mathbb{H}_{\mathcal{F},t+1}^{f} \right]. \tag{22}$$

Additionally, in equilibrium, the value of being unemployed, equation (21), equals the value of being informal, equation (20). This condition implies:

$$p\left(\theta_t\right) E_t \Gamma_{t,t+1} \mathbb{H}_{\mathcal{F},t+1}^{f} + \frac{\varphi}{u'\left(c_t\right)} = w_t^{\imath}. \tag{23}$$

The opportunity cost of being in the informal sector, which is equal to the sum of the expected value of searching for a job in the formal sector plus the labor disutility, equals the labor income in this sector.

3 Efficient and flexible wage equilibrium

For comparative purposes, and before determining formal wages in the decentralized economy, I consider both the equilibrium of the model under flexible wages, hereafter referred to as the flexible wage equilibrium, and the social planner solution that is referred to as the efficient equilibrium.

3.1 Efficient equilibrium

In this section, I consider the social planner solution. The efficient allocation will be the benchmark relative to which monetary policy results will be evaluated.

In a scenario of perfect competition in goods and labor markets, the social planner chooses the state-contingent path of c_t, l_t^f, $l_t^\imath$ and v_t to maximize the following joint welfare of households and managers:

$$U_t = E_t \sum_{t=1}^{\infty} \beta^t \left(u\left(c_t\right) - \varphi\left(l_t^f + l_t^\imath\right) - \frac{\kappa}{2}\mathcal{F}_t^2 l_t^f \right),$$

subject to the law of motion of employment, $l_{t+1}^f = (1-\rho)\, l_t^f + m(v_t, l_t^u)$, and aggregate resource constraints: $1 = l_t^u + l_t^f + l_t^\imath$, and $y_t = c_t$,

with $y_t = \left[\left(y_t^f\right)^{\frac{\gamma-1}{\gamma}} + \left(y_t^\imath\right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$, and $m(v_t, l_t^u) = \mathbb{N}\left(v_t\right)^{1-\mu}\left(l_t^u\right)^{\mu}$.

The first-order conditions with respect to v_t, l_{t+1}^f and $l_t^\imath$ are given, respectively, by:

$$\kappa \left(\frac{v_t}{l_t^f} \right) = \Upsilon_t^f m_v\left(v_t, l_t^u\right), \tag{24}$$

$$\Upsilon_t^f = \beta E_t \left[u'(c_{t+1}) \frac{\partial y_{t+1}}{\partial y_{t+1}^f} \frac{\partial y_{t+1}^f}{\partial l_{t+1}^f} - \varphi + \frac{\kappa}{2} \left(\frac{v_{t+1}}{l_{t+1}^f} \right)^2 + \Upsilon_{t+1}^f \left((1-\rho) - m_{l^u}\left(v_{t+1}, l_{t+1}^u\right) \right) \right], \tag{25}$$

$$u'(c_t) \frac{\partial y}{\partial y^\imath} \frac{\partial y_t^\imath}{\partial l_t^\imath} - \varphi = \Upsilon_t^f m_{l^u}\left(v_t, l_t^u\right), \tag{26}$$

where $m_v(v_t, l_t^u) = (1-\mu)\, q(\theta_t)$, $m_{l^u}(v_t, l_t^u) = \mu p(\theta_t)$, and $p(\theta_t) = \theta_t q(\theta_t)$.

Υ_t^f represents the *social value* of an additional worker in the formal sector. From equation (26), the *social value* of an additional worker in the informal sector $\Upsilon_t^\imath$ is equal to $u'(c_t)\frac{\partial y}{\partial y^\imath}\frac{\partial y_t^\imath}{\partial l_t^\imath} - \varphi$.

Reorganizing and combining equations (24), (25) and (26), I obtain the following efficient job creation condition (the algebra is provided in Appendix B4):

$$\frac{\kappa \mathcal{F}_t}{q(\theta_t)} = \beta E_t \left[(1-\mu)\, u'(c_{t+1}) \left(\frac{\partial y_{t+1}}{\partial y_{t+1}^f} mpl_{t+1}^f - \frac{\partial y_{t+1}}{\partial y_{t+1}^\imath} mpl_{t+1}^\imath + \frac{\kappa \mathcal{F}_{t+1}^2}{2u'(c_{t+1})} \right) + (1-\rho)\, \frac{\kappa \mathcal{F}_{t+1}}{q(\theta_{t+1})} \right]. \tag{27}$$

In what follows, I denote $\hat{x}_t$ as the log deviation of variable x_t from its steady-state value x.

To gain some intuition, I derive the log-linear version of equations (24) and (26):

$$\frac{2}{\rho(1-\mu)} s_v \left(\mu \hat{\theta}_t + \hat{\mathcal{F}}_t \right) = \Upsilon^f \hat{\Upsilon}_t^f, \tag{28}$$

$$\frac{\mu 2 s_v}{(1-\mu)} \left(\hat{\mathcal{F}}_t + \hat{\theta}_t \right) = \Upsilon^\imath \hat{\Upsilon}_t^\imath, \tag{29}$$

where $s_v = \frac{\frac{\kappa}{2}\mathcal{F}^2 l^f}{u'(c)c}$, and $\Upsilon^\imath = u'(c)\frac{\partial y}{\partial y^\imath}\frac{\partial y^\imath}{\partial l^\imath} - \varphi$.

Equation (28) implies that the *social value* of an additional job in the formal sector $\Upsilon^f \hat{\Upsilon}_t^f$ equals the marginal cost for a formal firm of adding a new worker. Similarly, equation (29) implies that the *social value* of an additional worker in the informal sector $\Upsilon^\imath \hat{\Upsilon}_t^\imath$ equals the *social value* of an additional unemployed worker. These two equations, together with the efficient job creation condition, are the benchmark relative to which the flexible wage equilibrium and the staggered wage bargained equilibrium will be compared.

3.2 Equilibrium under flexible wages

In this section, I derive the three main equations of the model that govern labor market dynamics outside the steady state, assuming that wages are flexible.

In an environment with search and matching frictions, wages are determined through a negotiation process between firms and workers. Once wages are set, firms choose the level of employment that maximizes their benefit. I assume that firms renegotiate their nominal wages every period according to the Nash Bargaining Solution. The conventional sharing rule implies:

$$(1-\phi)\,\mathbb{H}^f_{it} = \phi J^f_{it}, \tag{30}$$

where ϕ measures the worker's relative bargaining power. $\mathbb{H}^f_{it}$ and J^f_{ti} are the worker's and firm's surplus, defined in equation (22) and (9) respectively.

Then, replacing the expressions for $\mathbb{H}^f_t$ and J^f_t, and equation (23) into equation (30), I find that under a flexible wage-setting, all firms in the formal sector set the following real wage every period (see Appendix B3 for the derivation):

$$w^f_t = w^o_t = \quad \phi\left(p^f_t mpl^f_t + \tfrac{\kappa}{2}\tfrac{\mathcal{F}^2_t}{u'(c_t)} + \tfrac{\kappa\mathcal{F}_t}{u'(c_t)}\theta_t\right) + (1-\phi)\left(\tfrac{\varphi}{u'(c_t)}\right)\ . \tag{31}$$

The negotiated wage is a combination of what a worker contributes to the match and what the worker loses by accepting a job, weighted by relative bargaining power. When all formal wages are renegotiated every period, all formal firms set the same wage. This is why the subscript i disappears.

From the equilibrium condition $Q^u_t = Q^i_t$ in equation (23), combined with equation (8), gives:

$$\frac{\kappa\mathcal{F}_t\theta_t}{u'(c_t)}\frac{\phi}{1-\phi} + \frac{\varphi_t}{u'(c_t)} = w^\imath_t. \tag{32}$$

Replacing (32) into (31), I obtain an expression for the average formal wage as a linear combination between the firm's income from having a job filled and the outside option for workers:

$$w^f_t = w^o_t = \quad \phi\left(p^f_t mpl^f_t + \tfrac{\kappa}{2}\tfrac{\mathcal{F}^2_t}{u'(c_t)}\right) + (1-\phi)\left(w^\imath_t\right). \tag{33}$$

Different from the case without informality, the outside option for workers depends on wages in the informal sector $w^\imath_t$. Therefore, after an adverse aggregate productivity shock, wages in the informal sector decrease, decreasing the outside option for formal workers and, therefore, increasing the negative effect of the shock on formal wages. Additionally, the effect of a productivity shock on labor market tightness will also be exacerbated by the presence of informality. Indeed, the informal sector works as a buffer that absorbs workers in bad times, and vice versa. Therefore, after a negative productivity shock, a proportion of unemployed workers will find it more profitable to go to the informal sector, decreasing the probability of a formal vacancy being filled. This, in turn, pins down the firm's surplus and, therefore, the incentive to hire. The decrease in the firm's surplus will have an even larger impact on the hiring rate in the formal sector.

Finally, replacing equation (33) into equation (10), I obtain the following job creation condition:

$$\frac{\kappa \mathcal{F}_t}{q(\theta_t)} = \beta E_t \left[(1-\varphi) u'(c_{t+1}) \left(\frac{\partial y_{t+1}}{\partial y^f_{t+1}} mpl^f_{t+1} - \frac{\partial y_{t+1}}{\partial y^\imath_{t+1}} mpl^\imath_{t+1} + \frac{\kappa \mathcal{F}^2_{t+1}}{2u'(c_{t+1})} \right) + (1-\rho) \frac{\kappa \mathcal{F}_{t+1}}{q(\theta_{t+1})} \right], \tag{34}$$

where $mpl^\imath_{t+1}$ is the marginal productivity of labor in the informal sector. The total wholesale output is equal to $y_t = \left[\left(y^f_t\right)^{\frac{\gamma-1}{\gamma}} + (y^\imath_t)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$. Under flexible price and wage setting, prices in the formal and informal sectors are equal to the marginal increase in production due to one unit increase in y^f_t and in $y^\imath_t$ respectively, this is: $p^f_t = \frac{\partial y_t}{\partial y^f_t}$ and $p^\imath_t = \frac{\partial y_t}{\partial y^\imath_t}$.

Now, it is possible to compare the flexible wage equilibrium with the efficient equilibrium found in the previous subsection. Note that equation (27) is equivalent to (34) when $\mu = \phi$, which means the elasticity of the matching function with respect to vacancies $(1-\mu)$ is equal to the firm's bargaining power $(1-\phi)$. This is known as the Hosios condition, which is necessary to achieve the constrained Pareto efficiency in an economy with search and matching frictions (Hosios, 1990).

In addition to assuming an optimal subsidy that eliminates the distortion caused by monopolistic competition, efficiency also requires that equation (27) holds, together with the elimination of the inefficient dispersion of prices $\triangle_t = 1$. The absence of price dispersion requires keeping the price level constant, which can be attained by a policy that stabilizes the marginal cost in the retail sector mc_t at the level consistent with the firm's desired mark-up.

This implies that when wages in the formal sector are negotiated every period and the steady state is efficient, a zero price inflation policy is optimal.

4 Equilibrium under wage rigidities in the formal sector

In this section, I determine the equilibrium conditions assuming wage rigidities in the formal sector. In line with Gertler and Trigari (2009) and Tomas (2008), I suppose staggered wage contracting, where every period, each firm in the formal sector has a fixed probability $(1-\omega^w)$ of renegotiating salaries. When the firm has the chance to renegotiate its nominal wage, it negotiates with both the existing workers and the new hires, so that all workers in the firm receive the same wage. For firms that cannot renegotiate wages, they will maintain the nominal wage from the previous period, and new hires will receive the same wage.

I denote W_{it}^{f*} as the nominal wage of a formal firm i that renegotiates its salary in period t. I assume that, in renegotiating firms, managers and workers split the match surplus as follows:

$$(1-\phi)\,\mathbb{H}^{f}\left(W_{it}^{f*}\right)=\phi J^{f}\left(W_{it}^{f*}\right). \tag{35}$$

For simplicity, I assume that in renegotiating firms, the match surplus is split in the same way as in period-by-period Nash bargaining[6].

I next characterize the relation between the contract wage W_{it}^{f*}, and the evolution of the average nominal wage W_{t}^{f} across workers in the formal sector, which is given by:

$$W_{t}^{f}=\int_{0}^{1}W_{it}^{f}di. \tag{36}$$

Given the constant returns to scale, all firms renegotiating wages face the same optimization problem and set the same contract wage, then $W_{it}^{f*}=W_{t}^{f*}$. Additionally, since firms that can renegotiate wages are randomly chosen, equation (36) can be expressed recursively as

$$W_{t}^{f}=(1-\omega^{w})W_{t}^{f*}+\omega^{w}W_{t-1}^{f}. \tag{37}$$

Finally, the aggregate job creation condition can be expressed as follows:

$$\frac{\kappa\mathcal{F}_{t}}{q\left(\theta_{t}\right)}=\beta E_{t}\Gamma_{t,t+1}\left[u'(c_{t+1})\left(p_{t+1}^{f}mpl_{t+1}^{f}-\frac{W_{t+1}^{f}}{P_{t+1}}+\frac{\kappa\mathcal{F}_{t+1}^{2}}{2u'(c_{t+1})}\right)+(1-\rho)\frac{\kappa\mathcal{F}_{t+1}}{q\left(\theta_{t+1}\right)}\right]. \tag{38}$$

The job creation condition is the same as in the case of the flexible wage scenario. As such, under wage rigidities, efficiency requires that the Hosios condition holds, together with the elimination of the inefficient dispersion of prices and wages.

[6]The other option is to maximize the weighted average of the firm and worker surplus (see Gertler and Trigari, 2009). The Nash bargaining solution is given by:

$$\left[1-\chi_{t}(W_{it}^{f*})\right]\mathbb{H}_{t}^{f}(W_{it}^{f*})=\chi_{t}(W_{it}^{f*})\mathbb{J}_{it},$$

where $\chi_{t}(W_{it}^{f*})=\phi/\left(\phi+(1-\phi)\,\frac{\mu_{t}(W_{it}^{f*})}{\epsilon_{t}}\right)$. ϵ_{t} is the cumulative discount factor that workers use to value the contract wage, while $\mu_{t}(W_{it}^{f*})$ is the cumulative discount for the firm. $\chi_{t}(W_{it}^{f*})$ is the relative share that depends not only on the bargaining power but also on the horizon over which the worker and the firm value the impact of the contract wage. According to Gertler and Trigari (2009), this is called the "horizon effect" that influences the bargained wage. Under this setting, firms account for the implications of the contract wage for future hires, but workers care about wages only during their time working at the firm. They find that, while the horizon effect is interesting from a theoretical perspective, it turns out not to be quantitatively important in their baseline calibration. Therefore, in the same line as Thomas (2008), I assume that managers and workers split the match surplus the same as in the flexible-wage equilibrium.

4.1 The linearized model

In this subsection, I derive the log-linear approximation of the rational expectations equilibrium around the efficient steady state. I start by deriving the log-linear version of the three central equations that govern labor market dynamics outside the steady state: the relation for formal wages, the job creation condition in the formal sector, and the equilibrium condition in the informal sector (the log-linear and the steady state equations are presented in Appendix B1).

In Appendix B5, I show that log-linearizing and combining equations (35), (22), (23) and (9) results in the following law of motion for the average real wage in the formal sector:

$$\hat{w}_t^f = \psi_o \hat{w}_t^o + \psi_1 E_t \left(\hat{w}_{t+1}^f + \hat{\pi}_{t+1}\right) + \psi_2 \left(\hat{w}_{t-1}^f - \hat{\pi}_t\right), \tag{39}$$

where

$$\hat{w}_t^o = \ \phi \left[\ \Upsilon_a \hat{a}_t + \Upsilon_{\mathcal{F}} \left(2\hat{\mathcal{F}}_t - \hat{u}'(c_t)\right)\ \right] + \ (1-\phi)\left(\Upsilon_{w^\imath} \hat{w}_t^\imath\right) \tag{40}$$

is the real formal wage that would arise under period-by-period Nash bargaining. $a = p^f mpl^f$, $\Upsilon_a = \frac{a}{w^f}$, $\Upsilon_{\mathcal{F}} = \frac{\kappa \mathcal{F}^2}{w^f u'(c)}$, $\Upsilon_{w^\imath} = \frac{w^\imath}{w^f}$, $\psi_o + \psi_1 + \psi_2 = 1$, and $\hat{a}_t = \hat{p}_t^f + \widehat{mpl}_t^f$. Due to staggered wage negotiation, the average formal wage $\hat{w}_t^f$ depends on its lagged value $\hat{w}_{t-1}^f$ as well as on the expected future wage $E_t \hat{w}_{t+1}^f$. Under a flexible wage-setting where $\omega^w = 0$, both ψ_1 and ψ_2 become equal to zero, and ψ_o equal to 1, thus $\hat{w}_t^f = \hat{w}_t^o$.

Additionally, log-linearizing the aggregate job creation condition in equation (38), results in:

$$\hat{\mathcal{F}}_t - \hat{q}\left(\theta_t\right) = \frac{1}{J^f}\left(a\hat{a}_{t+1} - w^f \hat{w}_{t+1}^f\right) + \Gamma \hat{\mathcal{F}}_{t+1} + \Gamma\left(1-\rho\right)\hat{q}\left(\theta_{t+1}\right) + E_t\left(a-w\right)\frac{1}{J^f}\hat{u}'(c_{t+1}). \tag{41}$$

Finally, log-linearizing the equilibrium condition in the informal sector, equation (23), yields (see Appendix B5 for details):

$$E_t \Gamma p\left(\theta\right) \mathbb{H}^f \left(\hat{\mathcal{F}}_t + \hat{\theta}_t - \hat{u}'(c_t) - \frac{\nabla \omega^w}{1-\omega^w} E_t\left[\hat{w}_{t+1}^f - \hat{w}_t^f + \hat{\pi}_{t+1}\right]\right) = w^\imath \hat{w}_t^\imath + \frac{\varphi}{u'(c)}\hat{u}'(c_t), \tag{42}$$

where $\nabla = \frac{\mu w^f}{J^f}\left(\frac{J^f \epsilon}{\mathbb{H}^f \mu} + 1\right)$.

To gain some intuition, I next express the job creation condition (41) and the equilibrium condition in the informal sector (42) in terms of the inefficient fluctuations of the marginal cost and in the formal wage gap, as follows (the algebra is presented in Appendix B6):

$$\frac{2s_v}{\rho(1-\mu)}\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_t\right) - \Upsilon^f \hat{\Upsilon}_t^f = \beta E_t\left[\left(\frac{y^f}{y}\right)^{\frac{\gamma-1}{\gamma}} \hat{mc}_{t+1} + \frac{s_w}{(1-\mu)}\left(\hat{w}_{t+1}^o - \hat{w}_{t+1}^f\right)\right], \tag{43}$$

and

$$\frac{\mu 2 s_v}{(1-\mu)}\left(\hat{\mathcal{F}}_t+\hat{\theta}_t\right)-\Upsilon^{\imath}\hat{\Upsilon}^{\imath}_t=\left(\frac{y^{\imath}}{y}\right)^{\frac{\gamma-1}{\gamma}}\frac{l^u}{l^{\imath}}\hat{mc}_t+\frac{\mu 2 s_v}{(1-\mu)}\frac{\nabla\omega^w}{1-\omega^w}E_t\left[\hat{\pi}_{wt+1}-\hat{\pi}_{t+1}\right], \tag{44}$$

where $s_w=\frac{w^f l^f}{y}$ is the steady state formal labor income share, $s_v=\frac{\frac{\kappa}{2}\mathcal{F}^2 l^f}{u'(c)c}$ is the vacancy posting cost in consumption units as a fraction of total consumption, and $\hat{\pi}_{wt}$ is the wage inflation in the formal sector. Υ^f_t is the *social value* of an additional job in the formal sector, defined in equation (28), and $\Upsilon^{\imath}$ is the *social value* of an additional worker in the informal sector, defined in equation (29). Note that the left-hand side (LHS) of equation (43) represents the difference between the marginal cost for a formal firm of adding a worker and the social value of an additional job in the formal sector. This difference depends on the expected fluctuations in the marginal cost and on the formal wage gap. In the same way, the LHS of equation (44) represents the difference between the *social value* of an unemployed worker and the *social value* of an additional worker in the informal sector. This difference also depends on the fluctuations in the marginal cost and in the formal wage gap.

The Phillips curve, wage inflation equation and IS curve

By log-linearizing and combining equation (13) and (15) it is possible to obtain the standard expression of the price inflation, known as the Phillips curve (see Appendix B7):

$$\hat{\pi}_t=\kappa_{px}\left(\hat{mc}_t\right)+\beta E_t\hat{\pi}_{t+1}, \tag{45}$$

where $\kappa_{px}=\frac{(1-\omega^p)(1-\omega^p\beta)}{\omega^p}$. $\hat{mc}_t$ denotes the log-deviation of the real marginal cost from its steady state value.

I next derive an expression for wage inflation in the formal sector. From equation (39) and average real wage dynamics $\hat{w}^f_t-\hat{w}^f_{t-1}=\hat{\pi}_{wt}-\hat{\pi}_t$, I obtain the following expression for wage inflation (see Appendix B8 for the derivation):

$$\hat{\pi}_{wt}=\frac{\psi_0}{\psi_2}\left(\hat{w}^o_t-\hat{w}^f_t\right)+\frac{\psi_1}{\psi_2}E_t\left(\hat{\pi}_{wt+1}\right), \tag{46}$$

where $\frac{\psi_0}{\psi_2}=\frac{1-\omega^w}{\omega^w\left(1+\left(1-\rho+\mu\frac{\rho}{Jf}\right)\omega^w\beta\phi\right)}$, and $\frac{\psi_1}{\psi_2}=\frac{(1-\rho)\phi}{1+\left(1-\rho+\mu\frac{\rho}{Jf}\right)\omega^w\beta\phi}$.

According to equation (46), wage inflation depends on the gap between the target and the actual average real wage $\left(\hat{w}^o_t-\hat{w}^f_t\right)$. The intuition behind this result is as follows: when the average formal wage in the economy $\hat{w}^f_t$ is below (above) its target level $\hat{w}^o_t$, renegotiating firms will tend to increase (decrease) their nominal wages, thus generating positive (negative) wage inflation. Consequently, an aggregate productivity shock in the economy will affect $\hat{w}_t^o$, and formal real wages will converge slowly toward their target levels. In this case, the gap $\left(\hat{w}_t^o - \hat{w}_t^f\right) \neq 0$ generates an inefficient wage dispersion that translates into a hiring rate dispersion in the formal sector.

Finally, by log-linearizing the Euler equation, equation (17), I obtain a standard expression for the IS curve:

$$\hat{y}_t = E_t\left(\hat{y}_{t+1}\right) - \sigma\left(i_t - E_t\hat{\pi}_{t+1}\right). \tag{47}$$

5 Optimal monetary policy

In this section, I analyze optimal monetary policy in an economy with informality. I first derive the second-order approximation of the welfare criterion, which will be the objective function in the central bank's optimal monetary policy problem. To keep the analysis simple, I assume that the steady state of this economy is efficient. It implies that the Hosios condition holds ($\mu = \phi$), and there is a subsidy to monopoly firms (financed by a lump-sum tax to the same firms) that eliminates monopoly distortion.

In Appendix B9, I show that the second-order approximation of the household's welfare can be written as follows:

$$\sum_{t=0}^{\infty}\beta^t U_t = -\sum_{t=0}^{\infty}\beta^t \frac{u'(c)c}{2} L_t + t.i.p,$$

with

$$L_t = \Psi_\pi \hat{\pi}_t^2 + \Psi_{\pi w}\hat{\pi}_{wt}^2 + \mathcal{L}_t^{l,h}, \tag{48}$$

$$\mathcal{L}_t^{l,h} = \left(\sigma^{-1} - 1\right)\hat{y}_t^2 + 2s_v\left[\mu\hat{\theta}_t^2 + \hat{\mathcal{F}}_t^2\right] + \Psi_{yf}\left(\hat{l}_t^f\right)^2 + \Psi_{yi}\left(\hat{l}_t^i\right)^2,$$

where $\Psi_\pi = \frac{\Theta}{\Upsilon}$, $\Psi_{\pi w} = s_v 2\frac{\hbar^2}{\Upsilon_w}$, $\hbar = \frac{\beta\omega^w s_w}{(1-\beta\omega^w)\frac{2}{\rho}s_v}$, $\Upsilon_w = \frac{(1-\omega^w)(1-\beta\omega^w)}{\omega^w}$, $\Psi_{yf} = \left(\frac{y}{y^f}\right)^{\frac{1-\gamma}{\gamma}}$, and $\Psi_{yi} = \left(\frac{y}{y^i}\right)^{\frac{1-\gamma}{\gamma}}$.

$\mathcal{L}_t^{l,h}$ measures the success of monetary policy in stabilizing output and labor market variables around their efficient steady-state value. $t.i.p$ are the terms independent of policy. Note that in the case of a logarithmic utility function, $\sigma = 1$, and taking into account that the steady state is efficient, the value of $\mathcal{L}_t^{l,h}$ does not depend on output, and the efficient allocation of employment remains constant after an aggregate productivity shock. In this specific case, the variables in L_t are measured in terms of deviations from their efficient values. Hereafter I assume $\sigma = 1$.

Equation (48) shows that welfare decreases with price and wage inflation volatility. Under this framework, price inflation causes inefficient dispersion of prices across retail firms, and wage inflation creates inefficient dispersion of wages across formal firms[7]. Welfare also decreases with labor market tightness volatility. Indeed, the composition of total production between formal and informal goods can be inefficient if labor market tightness in the formal sector differs from its efficient value. Additionally, since the utility cost of hiring is convex in hiring rates, dispersion in $\mathcal{F}$ increases the welfare cost involved in job creation in the formal sector. Finally, inefficient fluctuations in employment, and hence in unemployment, are an additional source of welfare losses.

The contribution of inflation volatility to welfare losses Ψ_π is increasing in the degree of price stickiness. Similarly, the contribution of wage inflation volatility to welfare losses is increasing in the degree of wage stickiness and the steady-state formal labor income share s_w. In the presence of an informal sector, the proportion of firms in the economy facing wage rigidities is lower, together with the formal labor income share. As a result, the contribution of wage inflation volatility to the welfare loss, relative to the contribution of price inflation, is lower for the case with informality. Consequently, in the presence of an informal sector, the optimal policy will result in a lower price inflation volatility for a given level of wage inflation volatility.

---

[7] I show in Appendix B5 the hiring rate can be expressed as follows:

$$E_t\left[\hat{\mathcal{F}}_{it} - \hat{\mathcal{F}}_t\right] = -\omega^w \overline{w} \frac{\mu}{Q^o}\left[\hat{W}_{it}^{f*} - \hat{W}_t^{f*}\right].$$

This implies that wage dispersion creates dispersion in hiring rates: $var_i\left(\hat{\mathcal{F}}_{it}\right) = \left(\omega^w w \frac{\mu}{J^f}\right)^2 var_i\left(\hat{W}_{it}^{f*}\right)$

$$var_i\left(\hat{\mathcal{F}}_{it}\right) = \left(\frac{\beta\omega^w s_w}{(1-\beta\omega^w)\frac{2}{\rho}s_v}\right)^2 var_i\left(\hat{W}_{it}^{f*}\right)$$

$$var_i\left(\hat{\mathcal{F}}_{it}\right) = \hbar^2 var_i\left(\hat{W}_{it}^{f*}\right)$$

Since the hiring rate cost is convex and is measured in terms of utility, dispersion in the formal hiring rate increases the welfare cost involved in aggregate job creation.

5.1 Policy trade-offs

From the Phillips curve in equation (45), the wage inflation in equation (46), the hiring rate condition in the formal sector, equation (43), and the equilibrium condition in the informal sector, equation (44), it is possible to analyze the trade-offs faced by the monetary authority in an economy with a dual labor market and price and wage rigidities.

I first consider equations (45) and (46):

$$\hat{\pi}_t = \kappa_{px}\left(\hat{mc}_t\right) + \beta E_t \hat{\pi}_{t+1}, \tag{49}$$

$$\hat{\pi}_{wt} = \frac{\psi_o}{\psi_2}\left(\hat{w}_t^o - \hat{w}_t^f\right) + \frac{\psi_1}{\psi_2} E_t\left(\hat{\pi}_{wt+1}\right). \tag{50}$$

According to equation (50), wage inflation depends on the gap between the target and the average real wage. In response to an aggregate productivity shock, the average real wage in the formal sector is affected, but not as much as its natural (target) wage. Because of the presence of wage rigidities, formal real wages will converge slowly toward their target levels. As a result, the gap $\left(\hat{w}_t^o - \hat{w}_t^f\right) \neq 0$ translates into a formal wage inflation that results in inefficient wage dispersion.

When price inflation is equal to zero, $\hat{\pi}_t = 0$, equation (39) can be expressed as

$$\hat{w}_t^f = \psi_o \hat{w}_t^o + \psi_1 E_t \hat{w}_{t+1}^f + \psi_2 \hat{w}_{t-1}^f. \tag{51}$$

Equation (51) implies that in response to a real shock in the economy, $\hat{w}_t^f$ differs from $\hat{w}_t^o$ when price inflation is equal to zero. Therefore, under a zero price inflation policy, the central bank is not able to close the gap between actual and desired wages in the formal sector. It follows that the central bank faces a trade-off between price inflation and wage inflation.

I next consider the job creation condition in the formal and informal sector, defined in equations (43) and (44), respectively, as follows:

$$\tfrac{2s_v}{\rho(1-\mu)}\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_t\right) - \Upsilon^f \hat{\Upsilon}_t^f = \beta E_t\left[\left(\tfrac{y^f}{y}\right)^{\frac{\gamma-1}{\gamma}} \hat{mc}_{t+1} + \tfrac{s_w}{(1-\mu)}\left(\hat{w}_{t+1}^o - \hat{w}_{t+1}^f\right)\right],$$

$$\frac{\mu 2s_v}{(1-\mu)}\left(\hat{\mathcal{F}}_t + \hat{\theta}_t\right) - \Upsilon^\imath \hat{\Upsilon}_t^\imath = \left(\frac{y^\imath}{y}\right)^{\frac{\gamma-1}{\gamma}} \frac{l^u}{l^\imath}\hat{mc}_t + \frac{\mu 2s_v}{(1-\mu)}\frac{\nabla\omega^w}{1-\omega^w} E_t\left[\hat{\pi}_{wt+1} - \hat{\pi}_{t+1}\right].$$

Efficiency requires that equations (28) and (29) hold. However, when either $\left(\hat{w}_t^o - \hat{w}_t^f\right) \neq 0$ or $\hat{mc}_t \neq 0$, the job creation condition in the formal and informal sectors is inefficient. Therefore, under this framework, both price and wage inflation generate a distortion in the formal hiring rate and the job creation condition in the informal sector, which translates into inefficient fluctuations in unemployment. As a consequence, the central bank also faces a trade-off between price inflation and unemployment.

Note that the larger the gap $\left(\hat{w}_t^o - \hat{w}_t^f\right)$, the greater the inefficiency fluctuations in the labor market variables. In response to an aggregate productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. This wage rigidity generates a gap between the average formal wage $\hat{w}_t^f$ and the target wage $\hat{w}_t^o$. Equation (40) shows that in the presence of an informal sector, the target wage $\hat{w}_t^o$ depends, apart from productivity, on the informal wage (the outside option) and on the hiring rate (which depends on labor market tightness). As I noted in section 3.2, after an aggregate productivity shock, the effect in both variables is higher than in the case without informality.

Consequently, for a given level of inflation, the wage gap is higher in the presence of an informal sector. On that account, the inefficient fluctuations in labor market variables such as labor market tightness, hiring rate, and unemployment are also higher. It follows that the trade-off between price inflation and unemployment faced by the central bank increases in the presence of an informal sector.

In summary, the existence of a large informal sector has two opposite implications for optimal monetary policy. On the one hand, the optimal policy results in a lower price inflation volatility for a given level of wage inflation volatility. This is because wages in the informal sector are flexible, and hence the proportion of firms in the economy facing wage rigidities is lower. On the other hand, in the presence of informality, wage inflation and unemployment volatility are higher. As a result, the central bank should use price inflation to avoid excessive unemployment volatility and excessive dispersion in the formal hiring rate. The aggregate effect on price inflation volatility would depend on which effect dominates.

5.2 Responses under optimal monetary policy and quantitative analysis

In this section, I use numerical methods to characterize optimal monetary policy with informality. For simplicity, I focus only on the volatility generated by exogenous aggregate productivity shocks[8].

5.2.1 Calibration

A summary of the calibration of the model is reported in Table 1 The model is calibrated at a quarterly frequency to reproduce some key metrics for the Colombian economy[9]. The first set of parameters corresponds to standard values in the real business cycle literature (RBC). I set the quarterly discount factor $\beta = 0.99$, which implies a quarterly real rate of interest of approximately 1%. I assume an intertemporal elasticity of substitution equal to 1, $\sigma = 1$. Similar to Restrepo-Echavarría (2014), Fernández and Meza (2015), and Alberola and Urrutia (2021) I set the elasticity of substitution between formal and informal inputs, γ, equal to 8. The markup of prices on marginal costs is assumed to be on average 20 percent. This amount is obtained by setting Θ equal to 6.

In addition, based on most of the existing literature, bargaining power has typically been set either to satisfy the Hosios (1990) condition or to achieve symmetric Nash bargaining, in which the surplus is equally shared, I set the worker's bargaining power parameter, ϕ, and the elasticity of matches with respect to vacancies, μ, equal to 0.5. This assumption ensures the efficiency of the equilibrium in the flexible version of the model (Hosios, 1990). Finally, I assume an average duration of wage contracts of one year, and price contracts of one semester, i.e., $\omega^w = 0.75$ and $\omega^p = 0.5$ respectively.

The second set of parameters, (ρ, κ, $\mathbb{N}$,φ, $z^\imath$, and z^f) are jointly calibrated so that the steady state of the model matches the long-term properties of the data: an unemployment rate of 11%, a share of informally employed workers equal to 41%, a probability of filling a vacancy of 0.894, the relative productivity of the informal sector equal to 0.635, and a job-finding rate in the formal sector of 0.1371[10]. I obtain the job destruction rate in the formal sector

[8] As noted in section 5, when the utility function is logarithmic (when $\sigma = 1$), the value of $\mathcal{L}_t^{l,h}$ does not depend on output, and the efficient allocation of employment remains constant after an *aggregate productivity shock*. In this specific case, the variables in L_t are measured in terms of deviations from their efficient values.

[9] I choose Colombia as a benchmark country given the weight and persistence of the informal sector, and the availability of information on labor market flows and wage differentials across sectors

[10] The size of the informal sector measures the share of the urban labor force working in the informal sector in Colombia. The rates of unemployment and informal employment are calculated using data from the Household Survey (GEIH for its acronym in Spanish) and taking into account the definition of informality used by the Colombian System of National Accounts (DANE for its acronym in Spanish). DANE's definition of informal employment includes the group of employees and employers working in firms with fewer than 10 workers, unpaid family workers, domestic household workers, and self-employed individuals who are not professionals or technicians. Data on the probability of filling a vacancy in the formal sector comes from Cardozo (2019), who estimates that the time to fill a vacancy in Colombia is approximately 1.35 months. It will imply a monthly probability of filling a vacancy of 0.528 (i.e., a quarterly probability of 0.894). Data on the differential in productivity between the formal and informal sectors is taken from Fernandez (2018), who finds that informal firms have between 54% and 73% of the formal firm's productivity. Finally, the probability that an unemployed worker finds a job in the formal sector is estimated using data from

$\rho = 0.0314$, the adjustment cost parameter $\kappa = 155.17$, the efficiency parameter of the matching function $\mathbb{N} = 0.35$, the fixed component of labor disutility $\varphi = 0.048$, and the labor productivity in the informal sector $z^{\imath} = 0.635$, with z^f normalized to 1.

Table 1. Parameters for the baseline economy

Description	Symbol	Value	Justification
Fixed parameters			
Discount rate	β	0.99	Standard values in the RBC literature
Intertemporal elasticity of substitution	σ	1	Standard values in the RBC literature
Elasticity of substitution formal-informal inputs	γ	8	Restrepo-Echavarría (2014)
Elasticity of substitution between varieties	Θ	6	Standard value in the NK literature
Bargaining worker's power	ϕ	0.5	Standard value in the literature
Elasticity of matches with respect to vacancies	μ	0.5	Standard value in the literature
Fraction of formal firms not changing wages	ω^w	0.75	Average wage contracts of one year
Fraction of retail firms not changing prices	ω^p	0.5	Average price contracts of one semester
Labor productivity in the formal sector	z^f	1	Normalized to one
Calibrated to steady-state moments			**Targets**
Job destruction rate in the formal sector	ρ	0.0314	Job finding rate in formal sector: 0.1371
Adjustment cost parameter	κ	155.17	Informal employment rate: 0.41
Efficiency parameter of the matching function	$\mathbb{N}$	0.35	Unemployment rate: 0.11
Fixed component of labor disutility	φ	0.0166	Probability of filling a vacancy : 0.894
Labor productivity in the informal sector	$z^{\imath}$	0.635	Productivity gap: $z^{\imath}/z^f = 0.635$
Calibrated to business cycle moments			**Targets**
Persistence of aggregate productivity	ρ_z	0.89	Standard deviation of unemployment: 0.017
Standard deviation of productivity shocks	σ^z	0.0062	Standard deviation of GDP: 0.00698

The rest of the parameters are associated with aggregate productivity shock and the Taylor Rule and are jointly calibrated to match the volatility of gross domestic product (GDP) and unemployment. I set the persistence of the aggregate productivity $\rho_z = 0.89$, and the standard deviation of aggregate productivity shock $\sigma^z = 0.0062$ to match the volatility of GDP and unemployment to 0.0068 and 0.017, respectively, under a Taylor rule of the form $\hat{i}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t$.

5.2.2 Optimal monetary policy

I begin the quantitative analysis by simulating the behavior of the decentralized economy when the central bank implements the optimal monetary policy in an economy with and

the GEIH that allows estimating the allocation of workers between occupations (formal, informal, and unemployed) as well as calculating the transition rate between them.

without an informal sector.

At time 0, the central bank chooses the state-contingent plan that minimizes:

$$\sum_{t=0}^{\infty} \beta^t W L_t = -\sum_{t=0}^{\infty} \beta^t \frac{u'(c)c}{2} L_t + t.p.i + \mathcal{O}^3 \quad ,$$

with

$$L_t = \Psi_\pi \hat{\pi}_t^2 + \Psi_{\pi w} \hat{\pi}_{wt}^2 + \mathcal{L}_t^{l,h},$$

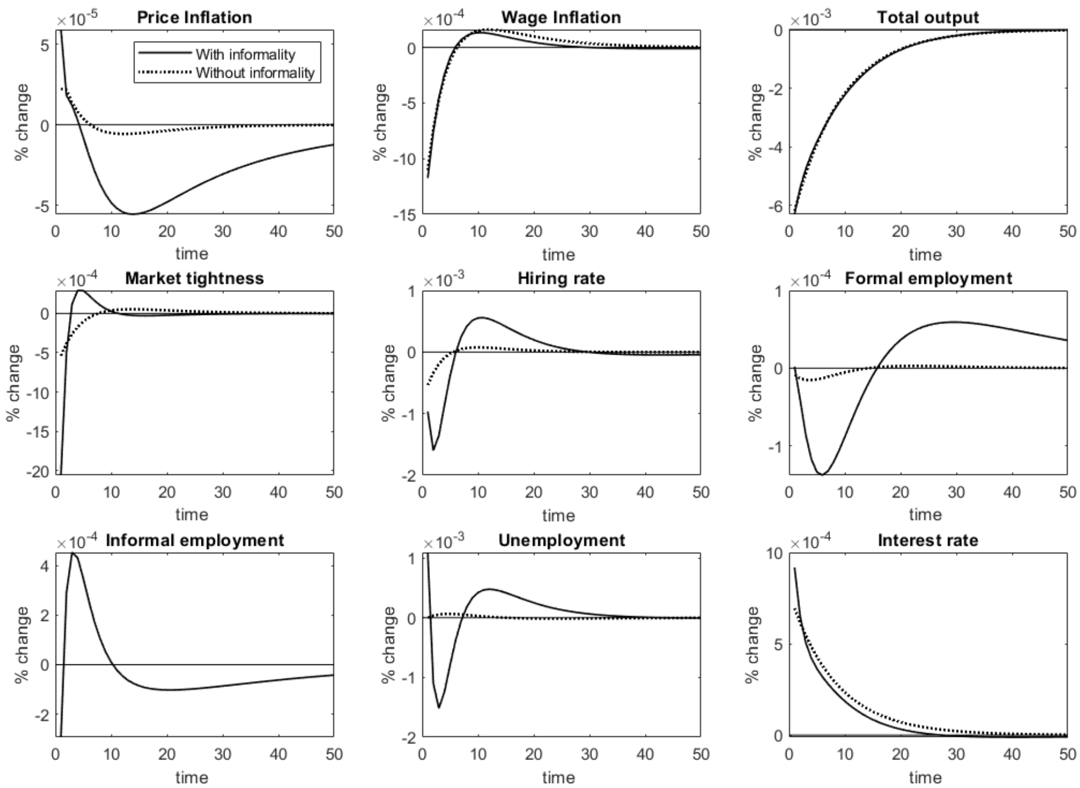
$$\mathcal{L}_t^{l,h} = \left(\sigma^{-1} - 1\right) \hat{y}_t^2 + 2 s_v \left[\mu \hat{\theta}_t^2 + \hat{\mathcal{F}}_t^2\right] + \Psi_{yf} \left(\hat{l}_t^f\right)^2 + \Psi_{yi} \left(\hat{l}_t^\imath\right)^2,$$

subject to the Phillips curve, equation (45), the law of motion of labor, equation (6), and the equilibrium condition in the informal sector, equation (44).

To better understand the effect of informality on optimal monetary policy design, I compare the predictions of the model to the case where there is no informal sector in the economy, in which case I assume $z^\imath = 0$. Figure 1 shows the Impulse Response Functions (IRFs) of all the variables in L_t, unemployment, and the interest rate, in response to one standard deviation ($\sigma^z = 0.0062$) negative productivity shock ($z_t \downarrow$) under the optimal monetary policy. Relative to the situation without informality, the optimal policy features significant deviations from price stability. After a negative productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. As a result, the gap between the target wage and the average wage in the formal sector decreases, which in turn generates negative wage inflation. By increasing the inflation rate, the central bank can affect the real value of nominal wages and then bring real formal wages closer to their flexible wage levels.

As noted in section 5.1, for a given level of price inflation, the wage gap is higher in the presence of an informal sector. On that account, the inefficient fluctuations in the labor market variables such as labor market tightness, hiring rate, and unemployment are also higher. It follows that the presence of an informal sector requires a higher adjustment on inflation to reduce this gap.

Figure 1. Impulse response functions to a 0.62% negative productivity shock under the optimal policy



From the welfare loss function, equation (48), I find that contribution of wage inflation volatility to the welfare loss, relative to the contribution of inflation $\frac{\Psi_{\pi w}}{\Psi_{\pi}}$, is lower in the case with informality. It implies that in the presence of an informal sector, the optimal policy results in a lower price inflation volatility for a given level of wage inflation volatility. However, I also find that for a given level of price inflation, the presence of an informal sector amplifies the effect of an aggregate productivity shock on wage inflation and on inefficient fluctuations in employment, implying that the central bank has to move further away from a full-price stabilization policy to reduce the formal wage gap. Figure 1 shows that under the baseline calibration, the second effect dominates, and the optimal policy features significant deviations from price stability in the presence of an informal sector [11].

[11] This result is robust to changes in the parameter values and model specifications. I repeat the same exercise with different values of the elasticity of substitution between formal and informal inputs, γ, different combinations of price and wage rigidities, and different values of the job destruction rates in the formal sector, ρ. I also develop different versions of the model: (i) with effort and changes in employment at the intensive and extensive margin and (ii) with search and matching friction in the informal sector. For all cases, I found that optimal policy features significant deviations from price stability in the presence of informality. The first effect dominates only for the case when both sectors are independent of each other and wages in the formal sector do not depend directly on informal labor market variables.

Table 2 shows the relative standard deviation (relative to the standard deviation of output) of price and wage inflation, output, employment, and unemployment under the optimal monetary policy, relative to the case without informality. The optimal volatility of price inflation is approximately three times higher for the case with informality. This result suggests that for emerging countries characterized by the presence of a large informal labor market, it is optimal to allow more inflation volatility.

Table 2. Relative standard deviations under Optimal monetary policy: with and without informality

	$l^{\imath} = 0.41$	$l^{\imath} = 0$
Standard Deviations[⊛]		
Price Inflation	0.0116	0.0035
Wage inflation	0.1769	0.1360
Output	0.0078	0.0110
Formal employment	0.0278	0.0032
Informal Employment	0.1003	-
Unemployment	0.3442	0.0138

[⊛] The standard deviation of output is expressed in absolute terms. The standard deviation of all other variables is divided by the standard deviation of output.

5.2.3 Zero price inflation and the Taylor Rule policy

To illustrate the implications of the trade-offs faced by the central bank, I analyze the behavior of the decentralized economy when the monetary authority implements a zero price inflation policy and the Taylor Rule.

Figure 2 plots the response of the economic variables to a 0.62% negative productivity shock under a zero price inflation policy. The decrease in aggregate productivity reduces the target wage in the formal sector, $\hat{w}_t^o$, via a fall in the marginal product of labor and a reduction in informal wages and labor market tightness. Note that under this policy, relative to the situation under the optimal policy, the decrease in wage inflation and the fluctuations in the rest of the labor market variables are much higher for the case with informality. Notice that for the case with informality unemployment decreases due to the larger increase in informal employment.

Figure 2. Impulse response functions to a 0.62% negative productivity shock under a zero price inflation policy

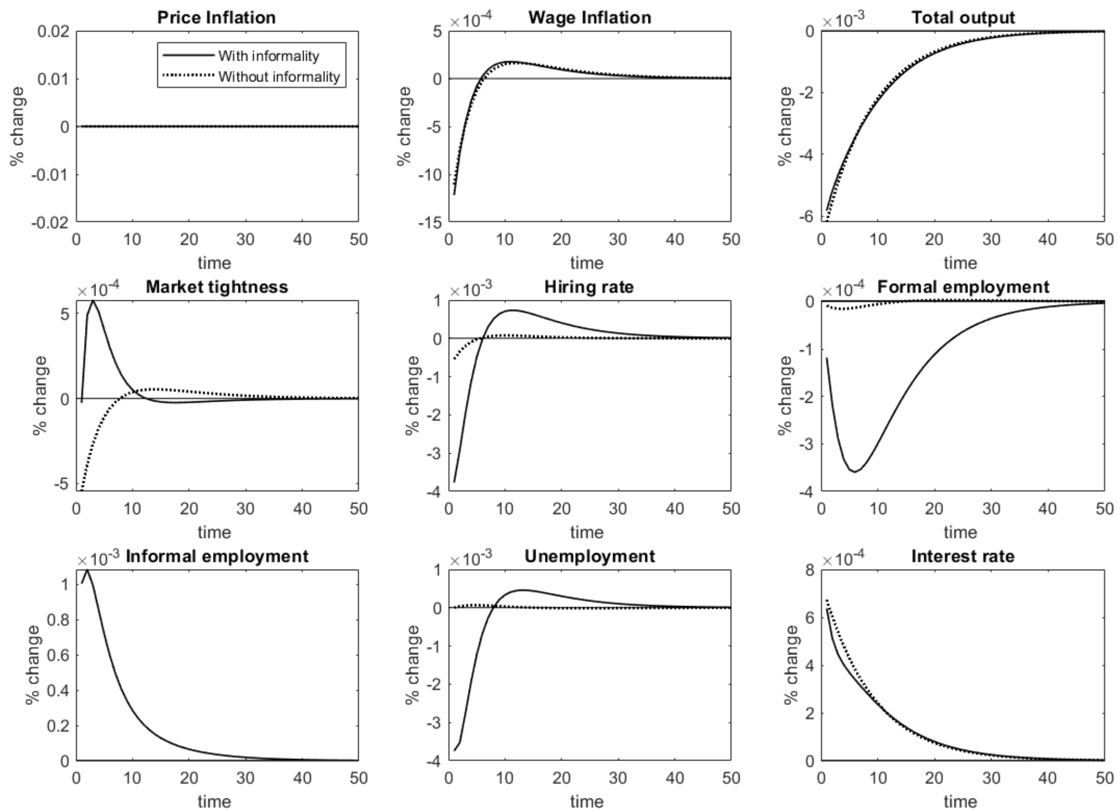


Table 3 shows the relative standard deviation of wage inflation, output, employment, and unemployment under a zero inflation policy for the cases with and without informality. By comparing Table 3 and Table 2, it is also possible to notice that under a zero inflation policy, wage inflation, formal and informal employment, and unemployment are much more volatile than under the optimal monetary policy, especially for the case with informality. By allowing some price inflation, the central bank can significantly reduce inefficient fluctuations in the labor market variables.

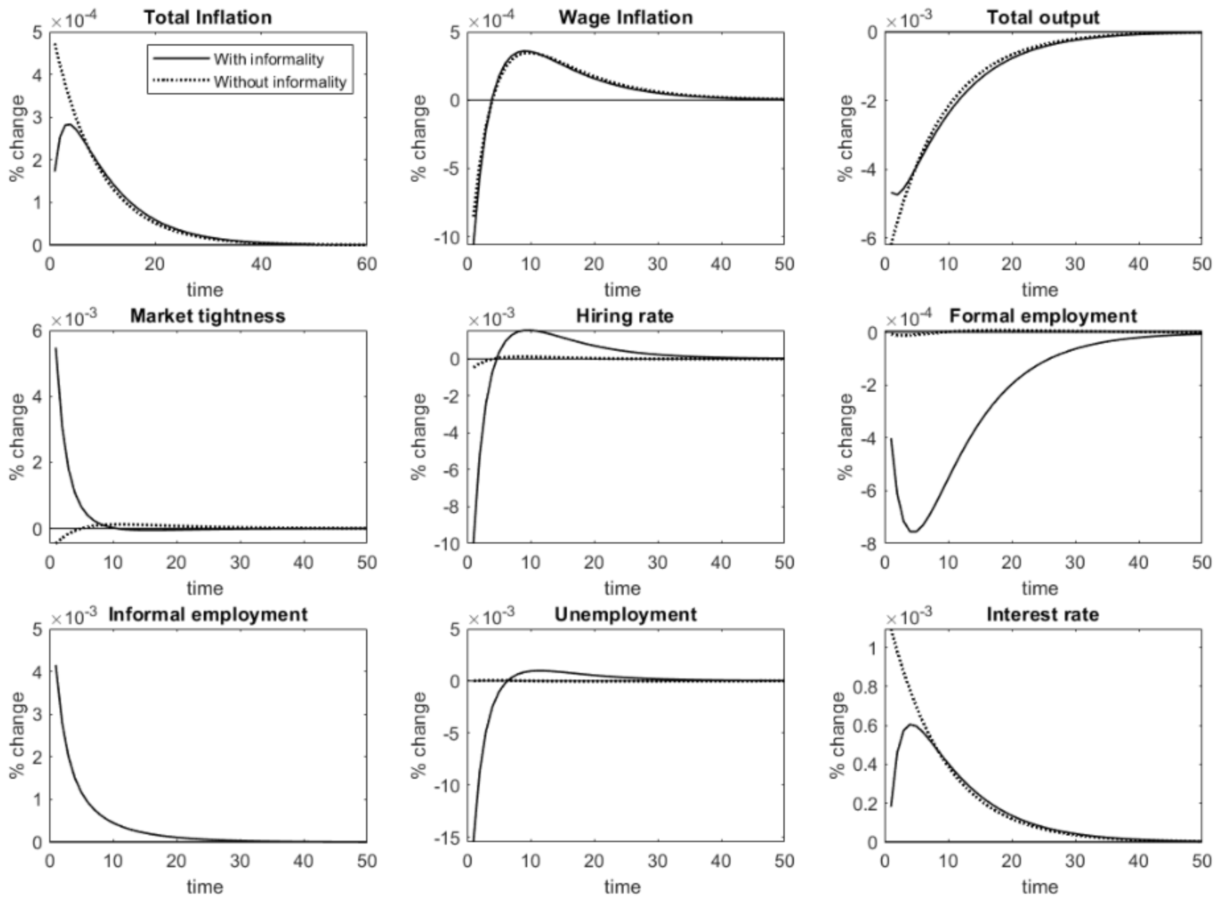
Table 3. Relative standard deviations under a zero price inflation policy

	$l^\imath = 0.41$	$l^\imath = 0$
Standard Deviations[⊛]		
Price inflation	0	0
Wage inflation	0.1941	0.1380
Output	0.0075	0.0110
Formal employment	0.0618	0.0033
Informal Employment	0.2034	-
Unemployment	0.7217	0.0141

[⊛] The standard deviation of output is expressed in absolute terms. The standard deviation of all other variables is divided by the standard deviation of output.

Additionally, I analyze the behavior of the decentralized economy when the central bank follows a Taylor Rule. Figure 3 shows the response of all the variables in the welfare loss function (L_t), unemployment, and the interest rate, to a 0.62% negative productivity shock under the Taylor Rule used in the benchmark calibration (i.e., $\hat{i_t} = 3.5\hat{\pi}_t + 0.09\hat{y}_t$). Relative to the situation without informality, the increase in price inflation is lower, while the decrease in wage inflation is higher. In both cases (with and without informality), output decreases and formal employment and the interest rate increase. Unemployment increases without informality and decreases in the presence of an informal sector. The decrease in unemployment is explained by the large increase in informal employment. Notice that all the labor market variables (formal employment, unemployment, wage inflation, hiring rate, etc.) are much more volatile in the presence of an informal sector.

Figure 3. Impulse response functions to a 0.62% negative productivity shock under the benchmark Taylor Rule*



*i.e., $\hat{i}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t$

Table 4 shows the relative standard deviation of price and wage inflation, output, employment, and unemployment for different Taylor Rules. The first rule considers a response of the interest rate to inflation of 3.5 and the response to output gap of 0.09 $\left(\hat{i} = 3.5\hat{\pi}_t + 0.09\hat{y}\right)$. The second rule considers a response of the interest rate only to inflation of 5 ($\hat{i} = 5\hat{\pi}_t$). Finally, the third Taylor Rule considers a response to inflation of 5 and to unemployment gap of 2 $\left(\hat{i}_t = 5\hat{\pi}_t + 2\hat{l_t^u}\right)$. In all cases, the volatility of unemployment and the rest of the labor market variables are higher in the presence of an informal sector. Particularly under the Taylor Rule that responds to inflation and output, the volatility of price and wage inflation, and the labor market variables are much higher for the case with informality. Therefore, in this model, a policy rule that targets output at the extent of wage inflation generates too much volatility in unemployment, especially for the case with informality. By targeting only price inflation, or price inflation along with unemployment, the central bank can considerably reduce labor market volatility.

Table 4. Relative standard deviations under different Taylor Rules and Optimal Policy

	$\hat{i}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t$		$\hat{i} = 5\hat{\pi}_t$		$\hat{i}_t = 5\hat{\pi}_t + 2\hat{l_t^u}$		**Optimal Policy**	
	$l^\iota = 0.41$	$l^\iota = 0$	$l^\iota = 0.41$	$l^\iota = 0$	$l^\iota = 0.41$	$l^\iota = 0$	$l^\iota = 0.41$	$l^\iota = 0$
Standard Deviations								
Price inflation	0.0576	0.0764	0.0199	0.0266	0.1516	0.0229	0.0116	0.0035
Wage inflation	0.1998	0.1437	0.1928	0.1592	0.1020	0.1598	0.1769	0.1360
Output	0.0068	0.0079	0.0072	0.0079	0.0078	0.0079	0.0078	0.0110
Formal employment	0.1510	0.0028	0.0894	0.0030	0.0197	0.0030	0.0278	0.0032
Informal employment	0.6698	-	0.3458	-	0.0817	-	0.1003	-
Unemployment	2.4997	0.0119	1.2699	0.0126	0.3019	0.0126	0.3442	0.0138

⊛ The standard deviation of output is expressed in absolute terms. The standard deviation of all other variables is divided by the standard deviation of output.

5.2.4 Welfare loss analysis

In line with Tomas (2008), I also consider a simple targeting rule that stabilizes a weighted average of price and wage inflation, with the same relative weights as in the welfare loss function. It writes:

$$\frac{\Psi_\pi}{\Psi_\pi + \Psi_{\pi w}}\hat{\pi}_t + \frac{\Psi_{\pi w}}{\Psi_\pi + \Psi_{\pi w}}\hat{\pi}_{wt} = 0.$$

Table 5 shows that any deviation from the optimal monetary policy under commitment generates higher welfare losses in the presence of an informal sector. A zero price inflation policy induces a substantial welfare cost under a staggered wage setting and in the presence of an informal sector, due to the excessive variation in wage inflation and unemployment. The welfare loss under the zero inflation policy is approximately 1.26 times as large as under the optimal policy, while for the case without informality the welfare loss under zero inflation is only approximately 0.015 times as large as under the optimal policy.

Different from Thomas (2008), who finds that the targeting rule that stabilizes a weighted average of price and wage inflation performs almost as well as the optimal policy, I find that in the presence of an informal sector, the same targeting rule generates significant welfare losses. By comparing equations (43) and (44), one can observe that the weighted average of price and wage inflation in the RHS of equation (43) is not equal to the weighted average of price and wage inflation in the RHS of equation (44). As a result, a simple targeting rule that stabilizes a weighted average of price and wage inflation is not enough to stabilize the formal hiring rate and informal employment at the same time.

The last two columns of Table 5 show that for the case where there are not wage rigidities, $\omega^w = 0$, and the economy's steady state is efficient, the central bank can replicate the efficient equilibrium with a full price inflation stabilization policy, even in the presence of informality.

Table 5. Welfare losses due to deviations from the optimal policy

	$\omega^w = 0.75$		$\omega^w = 0$	
Monetary policy	$l^\iota = 0.41$	$l^\iota = 0$	$l^\iota = 0.41$	$l^\iota = 0$
$\hat{\pi}_t = 0$	1.2579	0.0146	0	0
$\frac{\Psi_\pi}{\Psi_\pi + \Psi_{\pi w}} \hat{\pi}_t + \frac{\Psi_{\pi w}}{\Psi_\pi + \Psi_{\pi w}} \hat{\pi}_{wt} = 0$	1.2003	0.0063	-	-

6 Conclusions

This paper analyzes the optimal monetary policy in the presence of informality. I develop a closed economy model with nominal price and wage rigidities, search and matching frictions, and a dual labor market. I found that in the absence of wage rigidities and under an efficient steady state a zero price inflation policy is optimal. In a more realistic scenario, where both price and formal wage rigidities are present, a trade-off between inflation and unemployment emerges. I compare the predictions of the model against the case in which there is no informal sector in the economy. I found that the trade-off between price inflation and unemployment increases with the presence of an informal sector.

Under this framework, optimal monetary policy with informality features significant deviations from price stability in response to productivity shocks. In the presence of informality, wage inflation is more responsive to productivity shocks. Higher wage inflation generates a higher dispersion on wages in the formal sector. This wage dispersion translates into inefficient fluctuations in formal and informal employment and, thus, on unemployment. Therefore, by controlling the price inflation rate, the central bank is able to affect the real value of nominal wages and then bring real formal wages closer to their flexible wage levels. The presence of an informal sector requires a higher adjustment of inflation in order to reduce this gap.

To illustrate the implications of the trade-off faced by the central bank, I analyze the behavior of a decentralized economy when the monetary authority implements a policy of full inflation stabilization. I found that the welfare loss under the zero inflation policy is approximately 1.26 times as large as under the optimal policy, while for the case without informality, the welfare loss under the zero inflation policy is approximately 0.015 times as large as under the optimal policy. These results show that a policy designed to minimize inflation volatility can generate significant welfare losses in the presence of formal wage rigidities and informality, as is the case for most emerging countries.

Appendix B1 : Steady state and log-linearized equations

Steady state

$q\left(\theta\right)\mathcal{F}=\rho$

$m(v,\, l^{u})=\mathbb{N}\left(l^{u}\right)^{\mu}\left(v\right)^{1-\mu}$

$q\left(\theta\right)=\mathbb{N}\left(\theta\right)^{-\mu}$

$\frac{\kappa\mathcal{F}}{q(\theta)}=\beta E_{t}\left[u'(c)\left(p^{f}mpl^{f}-w^{f}\right)+\frac{\kappa}{2}\mathcal{F}^{2}+(1-\rho)\frac{\kappa\mathcal{F}}{q(\theta)}\right]$

$\frac{\kappa\mathcal{F}\theta}{u'(c)}=\left(w^{\imath}-\frac{\varphi}{u'(c)}\right)$

$w^{f}=\phi\left(p^{f}mpl^{f}+\frac{\kappa}{2}\frac{\mathcal{F}^{2}}{u'(c)}\right)+(1-\phi)\left(w^{\imath}\right)$

$w^{\imath}=p^{\imath}z^{\imath}z$

$y^{f}=zz^{f}l^{f}$

$y^{\imath}=zz^{\imath}l^{\imath}$

$y=\left[\left(y^{f}\right)^{\frac{\gamma-1}{\gamma}}+\left(y^{\imath}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$

$y^{f}=\left(\frac{1}{p^{f}}\right)^{\gamma}y$

$y^{\imath}=\left(\frac{1}{p^{\imath}}\right)^{\gamma}y$

$y=c$

log-linearized equations

$\hat{y}_{t}^{\imath}=\hat{z}_{t}+\hat{l}_{t}^{\imath}$

$\hat{w}_{t}^{\imath}=\hat{p}_{t}^{\imath}+\hat{z}_{t}$

$\hat{y}_{t}^{f}=\hat{z}_{t}+\hat{l}_{t}^{f}$

$\hat{a}_{t}=\hat{p}_{t}^{f}+\hat{y}_{t}^{f}-\hat{l}_{t}^{f}$

$\hat{y}_{t}=\hat{c}_{t}$

$\hat{y}_t^{\imath} = \gamma\left(\hat{mc}_t - p_t^{\imath}\right) + \hat{y}_t$

$\hat{y}_t^{f} = \gamma\left(\hat{mc}_t - p_t^{f}\right) + \hat{y}_t$

$\hat{y}_t = \Psi_{yf}\hat{y_t}^{f} + \Psi_{yi}\hat{y_t^{\imath}}.$ where $\Psi_{yf} = \left(\frac{y^f}{y}\right)^{\frac{\gamma-1}{\gamma}}$, $\Psi_{yi} = \left(\frac{y^{\imath}}{y}\right)^{\frac{\gamma-1}{\gamma}}$,

$\hat{m}_t = \mu\left(\hat{l}_t^{u}\right) + (1-\mu)\left(\hat{v}_t\right)$

$\hat{l}_{t+1}^{f} = \hat{l}_t^{f} + q\left(\theta\right)\mathcal{F}\left(\hat{q}\left(\theta_t\right) + \hat{\hat{\mathcal{F}}}_t\right)$

$\hat{q}\left(\theta_t\right) = \hat{m}_t - \hat{v}_t$

$\hat{p}\left(\theta_t\right) = (1-\mu)\,\hat{\theta}$

$\hat{\theta} = \hat{v}_t - \hat{l}_t^{u}$

$0 = l^u\hat{l}_t^{u} + l^{\imath}\hat{l}_t^{\imath} + l^f\hat{l}_t^{f}$

$\hat{\mathcal{F}}_t = \hat{v}_t - \hat{l}_t^{f}$

$0 = E_t\hat{\Gamma}_{t,t+1} + \hat{i}_t - \hat{\pi}_{t+1}$

$E_t\hat{\Gamma}_{t,t+1} = \hat{\lambda}_{t+1} - \hat{\lambda}_t$

$\hat{u}'(c) = -\frac{1}{\sigma}\hat{c}_t = \hat{\lambda}_t$

$\left(\hat{\mathcal{F}}_t - \hat{q}\left(\theta_t\right)\right) = \frac{1}{J^f}\left(a\hat{a}_{t+1} - w^f\hat{w}_{t+1}^{f}\right) + \Gamma\hat{\mathcal{F}}_{t+1} + \Gamma(1-\rho)\,\hat{q}\left(\theta_{t+1}\right) + E_t\left(a-w\right)\frac{1}{J^f}\hat{u}'(c_{t+1})$

$\hat{w}_t^{o} = \ \phi\left[\ \Upsilon_a\hat{a}_t + \Upsilon_{\mathcal{F}}\left(2\hat{\mathcal{F}}_t - \hat{u}'(c_t)\right)\ \right] + \ (1-\phi)\left(\varUpsilon_{w^{\imath}}\hat{w}_t^{\imath}\right)$

$E_t\Gamma p\left(\theta\right)\mathbb{H}^f\left(\hat{\theta}_t + \hat{\mathcal{F}}_t - \hat{u}'(c_t) - \frac{\omega^w\nabla}{1-\omega^w}E_t\left[\hat{w}_{t+1}^{f} - \hat{w}_t^{f} + \hat{\pi}_{t+1}\right]\right) = w^{\imath}\hat{w}_t^{\imath} + \frac{\varphi}{u'(c)}\hat{u}'(c)$

$\hat{w}_t^{f} = \psi_o\hat{w}_t^{o} + \psi_1 E_t\left(\hat{w}_{t+1}^{f} + \hat{\pi}_{t+1}\right) + \psi_2\left(\hat{w}_{t-1}^{f} - \hat{\pi}_t\right)$

$\hat{\pi}_t = \kappa_{px}\left(\hat{mc}_t\right) + \beta E_t\hat{\pi}_{t+1}$

$\hat{w}_t^{f} = \hat{w}_{t-1}^{f} + \hat{\pi}_{wt} - \hat{\pi}_t$

$\hat{z}_t^{f} = \rho_z^f\hat{z}_{t-1}^{f} + \varepsilon_t^{f}$

Appendix B2: Price settings

Total production of final goods in the informal sector, denoted with y_t^f is the following composite of individual retail goods:

$$y_t = \left[\int_o^1\left(y_{jt}^{\frac{\Theta-1}{\Theta}}\right)\right]^{\frac{\Theta}{\Theta-1}}$$

In the case that the firm has the chance to set prices optimally, it will choose the price that maximize the present discounted value of the firm's benefits, as follows:

$$\max_{p_t^*} \quad E_t\sum_{\ell=0}^{\infty}\varGamma_{t,t+\ell}\omega^{p\ell}\left[P_t^* y_{t+\ell/t} - MC_{t+\ell/t}y_{t+\ell/t}^{f}\right]$$

subject to the sequence of demand constraints:

$$y_{t+\ell/t} = \left(\frac{P_t^*}{P_{t+\ell}}\right)^{-\Theta} y_{t+\ell}. \tag{52}$$

The maximization problem can be written as follows

$$\max_{p_t^*} \quad E_t \sum_{\ell=0}^{\infty} \Gamma_{t,t+\ell} \omega^{p\ell} y_{t+\ell} \left[P_t^* \left(\frac{P_t^*}{P_{t+\ell}}\right)^{-\Theta} - (1-\tau^m) MC_{t+\ell/t} \left(\frac{P_t^*}{P_{t+\ell}}\right)^{-\Theta} \right]$$

The First order condition (FOC) with respect to p_t^* writes

$$E_t \sum_{\ell=0}^{\infty} \Gamma_{t,t+\ell} \omega^{p\ell} y_{t+\ell} \left[(1-\Theta) \left(\frac{P_t^*}{P_{t+\ell}}\right)^{-\Theta} + \Theta (1-\tau^m) MC_{t+\ell/t} \left(\frac{P_t^*}{P_{t+\ell}}\right)^{-1-\Theta} \frac{1}{P_{t+\ell}} \right] = 0$$

$$(P_t^*)^{1-\Theta} E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta} P_t^\Theta = \frac{\Theta(1-\tau^m)}{(1-\Theta)} (P_t^*)^{-\Theta} E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta} P_t^\Theta MC_{t+\ell}/t$$

$$P_t^* = \frac{\Theta(1-\tau^m)}{(1-\Theta)} \frac{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{-\Theta} \frac{MC_{t+\ell}}{P_{t+\ell}} P_t}{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta}}$$

Divining by P_t, and with $p_t^* = \frac{P_t^*}{P_t}$

$$p_t^* = \frac{\Theta(1-\tau^m)}{(1-\Theta)} \frac{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{-\Theta} mc_{t+\ell}}{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta}}$$

$$p_t^* = \frac{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{-\Theta} mc_{t+\ell}}{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{p\ell} \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta}}$$

Finally, the general price index in the formal sector is equal to:

$$P_t = \left(\omega^p (P_{t-1})^{1-\Theta} + (1-\omega^p)(P_t^*)^{1-\Theta}\right)^{\frac{1}{1-\Theta}}.$$

dividing both sides by P_t

$$1 = \left(\omega^p (\pi_t)^{1-\Theta} + (1-\omega^p)(p_t^*)^{1-\Theta}\right)^{\frac{1}{1-\Theta}}.$$

Appendix B3:Wage bargaining under a flexible wage setting

$$\max_{W_t^{f*}} \Phi_t = \left[J_{it}^f\right]^{1-\phi} \left[\mathbb{H}_{it}^f\right]^{\phi} \tag{53}$$

subject to:

$$W_{it}^f = \begin{cases} W_{it-1}^f & \textit{with probabiity}\ \omega^w \\ W_{it}^{f*} & \textit{with probabiity}\ (1-\omega^w) \end{cases} \tag{54}$$

J_{it}^f, and $\mathbb{H}_{it}^f$ can be expressed as follows:

$$\mathbb{H}_{it}^f = \frac{W_{it}^{f*}}{p_t} - \frac{\varphi_t}{u'(c)} + E_t\Gamma_{t,t+1}\left[(1-\rho)\mathbb{H}_{it+1}^f - p(\theta_t)\,\mathbb{H}_{\mathcal{F},t+1}^f\right] \tag{55}$$

$$J_{it}^f = a_t - w_{it}^f + \frac{\kappa\mathcal{F}_{it}^2}{2u'(c_t)} + (1-\rho)E_t\Gamma_{t,t+1}J_{it+1}^f \tag{56}$$

where

$$E_t\Gamma_{t,t+1}\left[p(\theta_t)\,\mathbb{H}_{\mathcal{F},t+1}^f\right] = w_t^{\imath} - \frac{\varphi_t}{u'(c)}$$

The first order necessary condition for the Nash bargaining solution is given by:

$$(1-\phi)\,\mathbb{H}_t^f(W_{it}^{f*}) = \phi J_t^f(W_{it}^{f*}) \tag{57}$$

Replacing (55) and (56) into (57), I obtain:

$$(1-\phi)\left(\frac{W_{it}^{f*}}{p_t} - \frac{\varphi_t}{u'(c)} + E_t\Gamma_{t,t+1}\left[(1-\rho-p(\theta_t))\mathbb{H}_{it+1}^f\right]\right) = \phi\left(a_t - w_{it}^f + \frac{\kappa\mathcal{F}_{it}^2}{2u'(c_t)} + (1-\rho)\frac{\kappa\mathcal{F}_t}{u'(c_t)q(\theta_t)}\right)$$

$$\frac{W_{it}^{f*}}{p_t} = \phi\left(a_t + \frac{\kappa\mathcal{F}_{it}^2}{2u'(c_t)} + \frac{\kappa\mathcal{F}_t\theta_t}{u'(c_t)}\right) + (1-\phi)\left(\frac{\varphi_t}{u'(c_t)}\right)$$

with $\left[\frac{\kappa\mathcal{F}_t\theta_t}{u'(c)}\frac{\phi}{1-\phi}\right] = w_t^{\imath} - \frac{\varphi_t}{u'(c)}$

$$\frac{W_{it}^{f*}}{p_t} = \phi\left(a_t + \frac{\kappa\mathcal{F}_{it}^2}{2u'(c_t)} + \frac{\kappa\mathcal{F}_t\theta_t}{u'(c)}\right) + (1-\phi)\left(w_t^{\imath} - \frac{\kappa\mathcal{F}_t\theta_t}{u'(c)}\frac{\phi}{1-\phi}\right)$$

$$\frac{W_{it}^{f*}}{p_t} = \phi\left(a_t + \frac{\kappa\mathcal{F}_{it}^2}{2u'(c_t)}\right) + (1-\phi)\left(w_t^{\imath}\right)$$

Appendix B4 . Efficient Equilibrium

The social planner chooses the state-contingent path of c, l^f, l^i and v_t to maximize the joint welfare of households and managers, subject to the law of motion of employment and the aggregate resource constraint:$l^f_{t+1} = (1-\rho)\, l^f_t + m(v_t, l^u_t)$, $1 = l^u_t + l^f_t + l^\imath_t$,and $y_t = c_t$.

with $y_t = \left[\left(y^f_t\right)^{\frac{\gamma-1}{\gamma}} + (y^\imath_t)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$ and $m(v_t, l^u_t) = \mathbb{N}\,(v_t)^{1-\mu}\,(l^u_t)^{\mu}$. The first-order conditions with respect to v_t, l^f_{t+1} and $l^\imath_t$ are given by

$$[v_t] \qquad \kappa\left(\frac{v_t}{l^f_t}\right) = \Upsilon^f_t m_v\left(v_t, l^u_t\right) \tag{58}$$

$$\left[l^f_{t+1}\right] \qquad \Upsilon^f_t = \beta\left[u'(c_{t+1})\frac{\partial y_{t+1}}{\partial y^f_{t+1}}\frac{\partial y^f_{t+1}}{\partial l^f_{t+1}} - \varphi + \frac{\kappa}{2}\left(\frac{v_{t+1}}{l^f_{t+1}}\right)^2 + \Upsilon^f_{t+1}\left((1-\rho) - m_{l^u}\left(v_{t+1}, l^u_{t+1}\right)\right)\right] \tag{59}$$

$$[l^\imath_t] \qquad u'(c_t)\frac{\partial y}{\partial y^\imath}\frac{\partial y^\imath_t}{\partial l^\imath_t} - \varphi = \Upsilon^f_t m_{l^u}\left(v_t, l^u_t\right) \tag{60}$$

where $m_v(v_t, l^u_t) = (1-\mu)\, q\,(\theta_t)$ and $m_{l^u}(v_t, l^u_t) = \mu p\,(\theta_t)$, $p\,(\theta_t) = \theta_t q\,(\theta_t)$ and $1-\mu = \frac{\partial m_t}{\partial v_t}\frac{v_t}{m_t}$. Υ^f_t is known as the social value of an additional worker in the formal sector.

reorganizing and replacing $\kappa\left(\frac{v_t}{l^f_t}\right)\frac{1}{(1-\mu)q^f_t} = \Upsilon^f_t$ and $u'(c_t)\frac{\partial y}{\partial y^\imath}\frac{\partial y^\imath_t}{\partial l^\imath_t} - \varphi = \Upsilon^f_t\,(m_2(v_t, l^u_t))$ into (59), I obtain the following expression for the efficient job creation condition:

$$\kappa\left(\frac{v_t}{l^f_t}\right)\frac{1}{(1-\mu)\,q^f} = \beta\left[u'(c_{t+1})\frac{\partial y_{t+1}}{\partial y^f_{t+1}} mpl^f_{t+1} - \varphi + \frac{\kappa}{2}\left(\frac{v_{t+1}}{l^f_{t+1}}\right)^2 + \Upsilon^f_{t+1}\left((1-\rho) + m_2(v_{t+1}, l^u_{t+1})(-1)\right)\right]$$

$$\kappa\left(\frac{v_t}{l^f_t}\right)\frac{1}{(1-\mu)\,q^f} = \beta\left[u'(c_{t+1})\frac{\partial y}{\partial y^f} mpl^f_{t+1} - \varphi + \frac{\kappa}{2}\left(\frac{v_{t+1}}{l^f_{t+1}}\right)^2 + \kappa\left(\frac{v_{t+1}}{l^f_{t+1}}\right)\frac{1}{(1-\mu)\,q\,(\theta_{t+1})}(1-\rho) - \left(u'(c_{t+1})\frac{\partial y}{\partial y^\imath}\frac{\partial y^\imath_{t+1}}{\partial l^\imath_{t+1}} - \varphi\right)\right]$$

$$\frac{\kappa\mathcal{F}_t}{q\,(\theta_t)} = \beta\left[(1-\mu)\,u'(c_{t+1})\left(\frac{\partial y_{t+1}}{\partial y^f_{t+1}} mpl^f_{t+1} - \frac{\partial y_{t+1}}{\partial y^\imath_{t+1}} mpl^\imath_{t+1} + \frac{\kappa\mathcal{F}^2_{t+1}}{2u'(c_{t+1})}\right) + (1-\rho)\,\frac{\kappa\mathcal{F}_{t+1}}{q\,(\theta_{t+1})}\right] \tag{61}$$

Appendix B5: Wage dynamics in the formal sector under staggered wage bargaining

The worker's surplus can be written as:

$$\begin{aligned}\mathbb{H}^f_t(w^{f*}_t) = \tfrac{W^{f*}_t}{p_t} - \varphi_t + E_t\Gamma_{t,t+1}\Big\{&(1-\rho)\mathbb{H}^f_{t+1}(w^{f*}_{t+1}) - p\,(\theta_t)\,\mathbb{H}^f_{\mathcal{F},t+1} \\ &+(1-\rho)\omega^w\left[\mathbb{H}^f_{t+1}(W^{f*}_t) - \mathbb{H}^f_{t+1}(W^{f*}_{t+1})\right]\Big\}\end{aligned} \tag{62}$$

the term $E_t\left[\mathbb{H}^f_{t+1}(W^{f*}_t)-\mathbb{H}^f_{t+1}(W^{f*}_{t+1})\right]$ writes as follows:

$$
\begin{gathered}
E_t\left[\mathbb{H}^f_{t+1}(W^{f*}_t)-\mathbb{H}^f_{t+1}(W^{f*}_{t+1})\right]=E_t\left[\frac{W^{f*}_t}{P_{t+1}}-\frac{W^{f*}_{t+1}}{P_{t+1}}\right]\\
+(1-\rho)\omega^w E_t\Gamma_{t,t+2}\left[\mathbb{H}^f_{t+2}(W^{f*}_t)-\mathbb{H}^f_{t+2}(W^{f*}_{t+1})\right]
\end{gathered}
$$

log-linearizing this equation and iterating forward, we have:

$$
\begin{gathered}
E_t\left[\hat{\mathbb{H}}_{t+1}(W^{f*}_t)-\hat{\mathbb{H}}^f_{t+1}(W^{f*}_{t+1})\right]\mathbb{H}^f=E_t w^f\left[\hat{W}^{f*}_t-\hat{W}^{f*}_{t+1}\right]\\
+(1-\rho)\omega^w\mathbb{H}^f\Gamma E_t\left[\hat{\mathbb{H}}_{t+2}(W^{f*}_t)-\hat{\mathbb{H}}^f_{t+2}(W^{f*}_{t+1})\right]
\end{gathered}
$$

$$
\begin{gathered}
E_t\left[\mathbb{H}^f_{t+1}(W^{f*}_t)-\mathbb{H}^f_{t+1}(W^{f*}_{t+1})\right]=\\
\frac{w}{\mathbb{H}^f}E_t\sum_0^\infty\left((1-\rho)\omega^w\Gamma\right)^i\left[\hat{W}^{f*}_t-\hat{W}^{f*}_{t+1}\right]
\end{gathered}
$$

$$
E_t\left[\hat{\mathbb{H}}^f_{t+1}(W^{f*}_t)-\hat{\mathbb{H}}^f_{t+1}(W^{f*}_{t+1})\right]=\frac{w^f\epsilon}{H^f}\left[\hat{W}^{f*}_t-\hat{W}^{f*}_{t+1}\right]
$$

with $w^f=\frac{W^{nf}}{P}$ and $\epsilon=\frac{1}{[1-\beta(1-\rho)\omega^w]}$

In this way, the complete log linearization of (62) takes the form:

$$
\begin{gathered}
\mathbb{H}^f\hat{\mathbb{H}}^f_t(w^{f*}_t)=w\hat{w}^{f*}_t-\hbar\hat{\hbar}_t+E_t\Gamma\left\{(1-\rho)\mathbb{H}^f\hat{\mathbb{H}}^f_{t+1}(w^{f*}_{t+1})-p(\theta)\,\mathbb{H}^f\left(\hat{p}(\theta_t)+\hat{\mathbb{H}}_{\mathcal{F},t+1}\right)\right.\\
\left.+(1-\rho)\omega^w\mathbb{H}^f\left[\hat{\mathbb{H}}^f_{t+1}(W^{f*}_t)-\hat{\mathbb{H}}^f_{t+1}(W^{f*}_{t+1})\right]\right\}+\left[(1-\rho)\mathbb{H}^f-p(\theta)\,\mathbb{H}^f\right]\hat{\Gamma}_{t,t+1}
\end{gathered}
$$

$$
\begin{gathered}
\hat{\mathbb{H}}^f_t(w^{f*}_t)=\frac{w^f}{\mathbb{H}^f}\left(\hat{w}^{f*}_t+(1-\rho)\omega^w\epsilon\Gamma\left[\hat{w}^{f*}_t-\hat{w}^{f*}_{t+1}-\hat{\pi}_{t+1}\right]\right)-\frac{\hbar}{\mathbb{H}^f}\hat{\hbar}_t\\
+E_t\Gamma\left\{(1-\rho)\left(\hat{\mathbb{H}}^f_{t+1}(w^{f*}_{t+1})+\hat{\Gamma}_{t,t+1}\right)-p(\theta)\left(\hat{p}(\theta_t)+\hat{\mathbb{H}}_{\mathcal{F},t+1}+\hat{\Gamma}_{t,t+1}\right)\right\}
\end{gathered}
$$

where $\hbar_t=\frac{\varphi_t}{u'(c_t)}$. With $p(\theta)\,\mathbb{H}^f\Gamma\left(\hat{p}(\theta_t)+\hat{\mathbb{H}}_{\mathcal{F},t+1}+\hat{\Gamma}_{t,t+1}\right)=\left(w^\imath\hat{w}^\imath_t-\hbar\hat{\hbar}_t\right)$, I have:

$$
\boxed{
\begin{gathered}
\hat{\mathbb{H}}^f_t(w^{f*}_t)=\frac{w^f}{\mathbb{H}^f}\left(\hat{w}^{f*}_t+(1-\rho)\omega^w\epsilon\Gamma\left[\hat{w}^{f*}_t-\hat{w}^{f*}_{t+1}-\hat{\pi}_{t+1}\right]\right)\\
-\frac{1}{\mathbb{H}^f}\left(w^\imath\hat{w}^\imath_t\right)+E_t\Gamma\left\{(1-\rho)\left(\hat{\mathbb{H}}^f_{t+1}(w^{f*}_{t+1})+\hat{\Gamma}_{t,t+1}\right)\right\}
\end{gathered}
} \tag{63}
$$

The producer surplus can be written as:

$$
J^f_t=a_t-\frac{w^{nf}_t}{p_t}-\frac{\kappa}{2}\mathcal{F}^2_t\frac{1}{u'(c)}+\left(1-\rho+q^f\mathcal{F}_t\left(W^{f*}_t\right)\right)E_t\Gamma_{t,t+1}J^f_{t+1}
$$

$$
\begin{gathered}
J^f_t\left(W^{f*}_t\right)=a_t-\frac{w^{nf}_t}{p_t}-\frac{\kappa}{2}\mathcal{F}^2_t\frac{1}{u'(c)}\\
+\left(1-\rho+q^f\mathcal{F}_t\left(W^{f*}_t\right)\right)E_t\Gamma_{t,t+1}\left[\omega^w J^f_{t+1}\left(W^{f*}_t\right)+(1-\omega^w)\,J^f_{t+1}\left(W^{f*}_{t+1}\right)\right]
\end{gathered}
$$

$$
\begin{gathered}
J^f_t\left(W^{f*}_t\right)=a_t-\frac{w^{nf}_t}{p_t}+\frac{\kappa}{2}\mathcal{F}^2_t\frac{1}{u'(c)}\\
+(1-\rho)\,E_t\Gamma_{t,t+1}J^f_{t+1}\left(W^{f*}_{t+1}\right)\\
+(1-\rho)\,\omega^w E_t\Gamma_{t,t+1}\left[J^f_{t+1}\left(W^{f*}_t\right)-J^f_{t+1}\left(W^{f*}_{t+1}\right)\right].
\end{gathered}
$$

The term $E_t\left[J^f_{t+1}(W^{f*}_t)-J^f_{t+1}(W^{f*}_{t+1})\right]$ can be written as follows:

$$\begin{gathered}E_t\left[J^f_{t+1}(W^{f*}_t)-J^f_{t+1}(W^{f*}_{t+1})\right]=\\-\left[\tfrac{W^{f*}_t}{p_t}\tfrac{p_t}{p_{t+1}}-\tfrac{W^{f*}_{t+1}}{p_{t+1}}\right]+\tfrac{\kappa}{2u'(c)}\left[\mathcal{F}_{t+1}(W^{f*}_t)^2-\mathcal{F}_{t+1}(W^{f*}_{t+1})^2\right]\\+\\+\omega^w\left(1-\rho\right)E_t\Gamma_{t,t+2}\left[J^f_{t+2}(W^{f*}_t)-J^f_{t+2}(W^{f*}_{t+1})\right]\end{gathered}\tag{64}$$

From $E_t\Gamma_{t,t+1}J^f_{t+1}=\frac{\kappa\mathcal{F}_t}{u'(c)q^f_t}$, I can obtain an expression for $\mathcal{F}_t\left(W^{f*}_t\right)-\mathcal{F}_t\left(W^{f*}_{t+1}\right)$

$$E_t\Gamma_{t,t+1}\omega^w\left[J^f_{t+1}\left(W^{f*}_t\right)-J^f_{t+1}\left(W^{f*}_{t+1}\right)\right]=\frac{\kappa}{u'(c_t)q^f_t}\left[\mathcal{F}_t\left(W^{f*}_t\right)-\mathcal{F}_t\left(W^{f*}_{t+1}\right)\right].\tag{65}$$

Replacing (65) into (64), and iterating forward I obtain:

$$E_t\left[\hat{J}^f_{t+1}(W^{f*}_t)-\hat{J}^f_{t+1}(W^{f*}_{t+1})\right]=-\tfrac{w^f\mu}{J^f}\left[\hat{W}^{f*}_t-\hat{W}^{f*}_{t+1}\right]$$

$$E_t\left[\hat{\mathcal{F}}_{t+1}\left(W^{f*}_t\right)-\hat{\mathcal{F}}_{t+1}\left(W^{f*}_{t+1}\right)\right]=-\omega^w\frac{w^f\mu}{J^f}\left[\hat{W}^{f*}_t-\hat{W}^{f*}_{t+1}\right]$$

where $\mu=\frac{1}{1-\omega^w\Gamma}$.

In this way the log-linearized version of the formal firm's and worker's surplus can be written, respectively, as follows

$$\begin{gathered}\hat{J}^f_t\left(w^{fn*}_t\right)=\tfrac{a}{J^f}\hat{a}_t-\tfrac{w^f}{J^f}\left[\hat{W}^{f*}_t-\hat{P}_t+\left(1-\rho\right)\Gamma\omega^w\mu E_t\left[\hat{w}^{f*}_t-\hat{w}^{f*}_{t+1}-\hat{\pi}_{t+1}\right]\right]+\tfrac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w^{f*}_t\right)-\hat{u}'(c)\right)\\+\left(1-\rho\right)\Gamma E_t\left[\hat{J}^f_{t+1}\left(w^{f*}_{t+1}\right)+\hat{\Gamma}_{t,t+1}\right]\end{gathered}$$

$$\begin{gathered}\hat{\mathbb{H}}^f_t\left(w^{f*}_t\right)=\tfrac{w}{H}\left(\hat{w}^{f*}_t+(1-\rho)\omega^w\epsilon\Gamma\left[\hat{w}^{f*}_t-\hat{w}^{f*}_{t+1}-\hat{\pi}_{t+1}\right]\right)\\+E_t\Gamma\left\{\left(1-\rho\right)\left(\hat{\mathbb{H}}^f_{t+1}(w^{f*}_{t+1})+\hat{\Gamma}_{t,t+1}\right)-p\left(\theta\right)\left(\hat{p}\left(\theta_t\right)+\hat{\mathbb{H}}_{\mathcal{F},t+1}+\hat{\Gamma}_{t,t+1}\right)\right\}\end{gathered}$$

where a_t is the marginal productivity of labor $\hat{a}_t=\hat{p}^x_t+\hat{y}^f_t-\hat{l}^f_t$.

The wage contract would be in the following way:

Managers and workers split the match surplus in the same way as in the case of period-by-period Nash negotiation:

$$\left(1-\phi\right)\mathbb{H}^f_t=\phi J^f_t$$

log-linearizing this equation gives

$$\hat{\mathbb{H}}_t^f(W_t^{f*}) = J_t^f(W_t^{f*}) \tag{66}$$

replacing the expression for $\hat{\mathbb{H}}_t^f(W_t^{f*})$ and $J_t^f(W_t^{f*})$

$$\begin{gathered}
\frac{w}{\mathbb{H}^f}\left(\hat{w}_t^{f*} + (1-\rho)\omega^w\epsilon\Gamma\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \hat{\pi}_{t+1}\right]\right) \\
-\frac{1}{\mathbb{H}^f}\left(w^\imath \hat{w}_t^\imath\right) + E_t\Gamma\left\{(1-\rho)\left(\hat{\mathbb{H}}_{t+1}^f(w_{t+1}^{f*}) + \hat{\Gamma}_{t,t+1}\right)\right\} \\
= \frac{a}{J^f}\hat{a}_t - \frac{w^f}{J^f}\left[\hat{w}_t^{f*} + (1-\rho)\,\Gamma\omega^w\mu E_t\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \hat{\pi}_{t+1}\right]\right] \\
+\frac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w_t^{f*}\right) - \hat{u}'(c)\right) \;\; + (1-\rho)\,\Gamma E_t\left[\hat{J}_{t+1}^f\left(w_{t+1}^{f*}\right) + \hat{\Gamma}_{t,t+1}\right]
\end{gathered}$$

replacing $\hat{\mathbb{H}}_{t+1}^f(W_{t+1}^{f*}) = \hat{\mathbb{Q}}_{t+1}^o(W_{t+1}^{f*})$ and $\frac{\phi}{1-\phi} = \frac{\mathbb{H}^f}{J^f}$

$$\begin{gathered}
\frac{w}{\mathbb{H}^f}\left(\hat{w}_t^{f*} + (1-\rho)\omega^w\epsilon\Gamma\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \pi_{t+1}\right]\right) \\
-\frac{1}{\mathbb{H}^f}\left(w^\imath \hat{w}_t^\imath\right) + E_t\Gamma\left\{(1-\rho)\left(\hat{\mathbb{H}}_{t+1}^f(w_{t+1}^{f*}) + \hat{\Gamma}_{t,t+1}\right)\right\} \\
= \frac{a}{J^f}\hat{a}_t - \frac{w^f}{J^f}\left[\hat{w}_t^{f*} + (1-\rho)\,\Gamma\omega^w E_t\mu\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \pi_{t+1}\right]\right] \\
+\frac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w_t^{f*}\right) - \hat{u}'(c)\right) + (1-\phi)^{-1}\,\hat{\phi}_t(W_t^{f*}) \\
+ (1-\rho)\,\Gamma E_t\left[\hat{\Gamma}_{t,t+1} + \hat{\mathbb{H}}_{t+1}^f(W_{t+1}^{f*}) - (1-\phi)^{-1}\,\hat{\phi}_{t+1}(W_{t+1}^{f*})\right]
\end{gathered}$$

then

$$\begin{gathered}
(1-\phi)\,w\left(\hat{w}_t^{f*} + (1-\rho)\omega^w\epsilon\Gamma\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \pi_{t+1}\right]\right) - \frac{(1-\phi)}{\mathbb{H}^f}\left(w^\imath \hat{w}_t^\imath\right) \\
= \phi a\hat{a}_t - w^f\phi\left[\hat{w}_t^{f*} + (1-\rho)\,\Gamma\omega^w E_t\mu\left(\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \pi_{t+1}\right)\right] \\
+\frac{\phi}{(1-\phi)}\frac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w_t^{f*}\right) - \hat{u}'(c)\right)
\end{gathered}$$

rearranging

$$\begin{gathered}
w\hat{w}_t^{f*} + \left((1-\phi)\,\epsilon + \phi\mu\right)(1-\rho)\omega^w\Gamma w^f\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \pi_{t+1}\right] \\
= (1-\phi)\left(w^\imath \hat{w}_t^\imath\right) + \phi a\hat{a}_t \\
+\phi\frac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w_t^{f*}\right) - \hat{u}'(c)\right)
\end{gathered}$$

$$\begin{gathered}
w\hat{w}_t^{f*} + \left((1-\rho)\omega^w\Gamma\right)\bar{\Theta}w^f E_t\left[\hat{w}_t^{f*} - \hat{w}_{t+1}^{f*} - \pi_{t+1}\right] \\
= \phi a\hat{a}_t + (1-\phi)\left(w^\imath \hat{w}_t^\imath\right) + \phi\frac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w_t^{f*}\right) - \hat{u}'(c)\right)
\end{gathered}$$

where$\bar{\Theta} = (1-\phi)\,\epsilon + \phi\mu$

$$\begin{gathered}
(1+\Psi)\,\hat{w}_t^{f*} - \Psi E_t\left[\hat{w}_{t+1}^{f*} + \pi_{t+1}\right] \\
= \frac{\phi}{w}a\hat{a}_t + \frac{(1-\phi)}{w}\left(w^\imath \hat{w}_t^\imath\right) + \frac{\phi}{w}\frac{\beta q^f\mathcal{F}}{2}\left(2\hat{\mathcal{F}}\left(w_t^{f*}\right) - \hat{u}'(c)\right)
\end{gathered}$$

$$\hat{w}_t^{f*} = \frac{1}{(1+\Psi)}\hat{w}_t^o + \frac{\Psi}{(1+\Psi)}E_t\left[\hat{w}_{t+1}^{f*} + \hat{\pi}_{t+1}\right] \tag{67}$$

where $\Psi = (1-\rho)\omega^w \Gamma \bar{\Theta}$ and $\hat{w}_t^o$ is the target wage given by:

$$\hat{w}_t^o = \ \phi \Big[\ \Upsilon_a \hat{a}_t + \Upsilon_{\mathcal{F}} \left(2\hat{\mathcal{F}}_t - \hat{u}'(c) \right) \ \Big] + \ (1-\phi)\left(\Upsilon_{w^\imath} \hat{w}_t^\imath \right)$$

with, $a = p^f mpl^f$, $\Upsilon_a = \frac{a}{w^f}$, $\Upsilon_{\mathcal{F}} = \frac{\kappa \mathcal{F}^2}{w^f u'(c)}$, $\Upsilon_{w^\imath} = \frac{w^\imath}{w^f}$, $\psi_o + \psi_1 + \psi_2 = 1$, and $\hat{a}_t = \hat{p}_t^f + \widehat{mpl}_t^f$.

Formal wage and Hiring dynamics

the target wage is

$$\hat{w}_t^o \left(w_t^{f*} \right) = \left[\Upsilon_a \hat{a}_t + \Upsilon_{w^\imath} \hat{w}_t^\imath + \Upsilon_{\mathcal{F}} \left(2\hat{\mathcal{F}} \left(w_t^{f*} \right) - \hat{u}'(c) \right) \right]$$

Let's find expressions for $\hat{\mathcal{F}} \left(w_t^{f*} \right)$, and $\hat{\mathbb{H}}_{\mathcal{F},t+1}$ in terms of gaps between contract and average wages. Previously, I found $E_t \left[\hat{\mathcal{F}}_{t+1} \left(W_t^{f*} \right) - \hat{\mathcal{F}}_{t+1} \left(W_{t+1}^{f*} \right) \right] = -\omega^w w^f \frac{\mu}{J^f} \left[\hat{W}_t^{f*} - \hat{W}_{t+1}^{f*} \right]$ then

$$E_t \left[\hat{\mathcal{F}}_t \left(W_t^{f*} \right) - \hat{\mathcal{F}}_t \left(W_t^f \right) \right] = -\omega^w w^f \frac{\mu}{J^f} \left[\hat{W}_t^{f*} - \hat{W}_t^f \right]$$

where $\hat{\mathcal{F}}_t \left(w_t^{nf} \right)$ is the average hiring rate

Using the results in previous section

$E_t \left[\hat{J}_{t+1}^f (W_t^{f*}) - \hat{J}_{t+1}^f (W_{t+1}^{f*}) \right] = -w^f \frac{\mu}{J^f} \left[\hat{W}_t^{f*} - \hat{W}_{t+1}^{f*} \right]$

$E_t \left[\hat{\mathbb{H}}_{t+1}^f (W_t^{f*}) - \hat{\mathbb{H}}_{t+1}^f (W_{t+1}^{f*}) \right] = \frac{w^f \epsilon}{\mathbb{H}^f} \left[\hat{W}_t^{f*} - \hat{W}_{t+1}^{f*} \right]$

$$E_t \left[\hat{J}_{t+1}^f (W_{t+1}^{f*}) - \hat{J}_{t+1}^f (W_{t+1}^f) \right] = -w^f \frac{\mu}{J^f} \left[\hat{W}_{t+1}^{f*} - \hat{W}_{t+1}^f \right]$$

$$E_t \left[\hat{\mathbb{H}}_{t+1}^f (W_{t+1}^{f*}) - \hat{\mathbb{H}}_{t+1}^f (W_{t+1}^f) \right] = \frac{w\epsilon}{\mathbb{H}^f} \left[\hat{W}_{t+1}^{f*} - \hat{W}_{t+1}^f \right]$$

with $\phi_w = \frac{w^f}{J^f}$

$$E_t \left[\hat{J}_{t+1}^f (W_{t+1}^{f*}) - \hat{J}_{t+1}^f (W_{t+1}^f) \right] = -\phi_w \mu \left[\hat{W}_{t+1}^{f*} - \hat{W}_{t+1}^f \right]$$

$$E_t \left[\hat{\mathbb{H}}_{t+1}^f (W_{t+1}^{f*}) - \hat{\mathbb{H}}_{t+1}^f (W_{t+1}^f) \right] = \phi_w \frac{1-\phi}{\phi} \epsilon \left[\hat{W}_{t+1}^{f*} - \hat{W}_{t+1}^f \right]$$

Starting from the Nash first order condition in $t+1$:

$$E_t \hat{\mathbb{H}}_{t+1}^f (W_{t+1}^{f*}) = E_t \hat{J}_{t+1}^f (W_{t+1}^{f*})$$

$$\hat{\mathbb{H}}^f_{t+1}(W^f_{t+1}) + \frac{w^f\epsilon}{\mathbb{H}^f}\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right] = \hat{J}^f_{t+1}(W^f_{t+1}) - \frac{w^f\mu}{J^f}\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right]$$

$$\hat{\mathbb{H}}^f_{t+1}(W^f_{t+1}) + \left(\frac{w^f\epsilon}{\mathbb{H}^f} + \frac{w^f\mu}{J^f} - \left(q^f\mathcal{F}\omega^w\Gamma\right)\left(\omega^w\mu\phi_w\right)\mu\right)\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right] = \hat{J}^f_{t+1}(W^f_{t+1})$$

$$\hat{\mathbb{H}}^f_{t+1}(W^f_{t+1}) + \nabla\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right] = \hat{J}^f_{t+1}(W^f_{t+1})$$

where $\nabla = \phi_w\mu\left(\frac{J^f\epsilon}{\mathbb{H}^f\mu} + 1\right)$. Using $E_t\Gamma_{t,t+1}J^f_{t+1} = \frac{\kappa\mathcal{F}_t}{u'(c)q^f_t}$

$$\left(\hat{\Gamma}_{t,t+1} + \hat{J}^f_{t+1}(W^{f*}_{t+1})\right) = \hat{\mathcal{F}}_t\left(W^{f*}_t\right) - \hat{u}'(c_t) - \hat{q}\left(\theta_t\right)$$

we have then:

$$\hat{\mathbb{H}}^f_{t+1}(w^{nf}_{t+1}) + \nabla\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right] = \hat{\mathcal{F}}_t\left(W^f_t\right) - \hat{u}'(c_t) - \hat{q}\left(\theta_t\right) - \hat{\Gamma}_{t,t+1} +$$

$$E_t\left[\hat{\mathbb{H}}^f_{t+1}(w^{nf}_{t+1}) + \hat{\Gamma}_{t,t+1}\right] = \hat{\mathcal{F}}_t\left(W^f_t\right) - \hat{u}'(c_t) - \hat{q}\left(\theta_t\right) - \nabla E_t\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right]$$

where $\hat{\mathbb{H}}^f_{t+1}(w^{nf}_{t+1}) = \hat{\mathbb{H}}_{\mathcal{F},t+1}$. Then

$$E_t\left[\hat{\mathbb{H}}^f_{\mathcal{F},,t+1} + \hat{\Gamma}_{t,t+1}\right] = \hat{\mathcal{F}}_t\left(W^f_t\right) - \hat{u}'(c_t) - \hat{q}\left(\theta_t\right) - \nabla E_t\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right]$$

substituting in the target wage and rearranging

$$\hat{w}^o_t = \ \phi\left[\ \Upsilon_a\hat{a}_t + \Upsilon_{\mathcal{F}}\left(2\hat{\mathcal{F}}_t - \hat{u}'(c)\right)\ \right] + \ (1-\phi)\left(\Upsilon_{w^\imath}\hat{w}^\imath_t\right)$$

$\left[\hat{\mathcal{F}}_t\left(W^{f*}_t\right) - \hat{\mathcal{F}}_t\left(W^f_t\right)\right] = -\omega^w w^f\frac{\mu}{J^f}\left[\hat{W}^{f*}_t - \hat{W}^f_t\right]$
$E_t\left[\hat{\mathbb{H}}^f_{\mathcal{F},,t+1} + \hat{\Gamma}_{t,t+1}\right] = \hat{\mathcal{F}}_t\left(W^f_t\right) - \hat{u}'(c_t) - \hat{q}\left(\theta_t\right) - \nabla E_t\left[\hat{W}^{f*}_{t+1} - \hat{W}^f_{t+1}\right]$

$$\hat{w}^o_t = \ \phi\left[\ \Upsilon_a\hat{a}_t + \Upsilon_{\mathcal{F}}\left(2\hat{\mathcal{F}}_t - \hat{u}'(c)\right)\ \right] + \ (1-\phi)\left(\Upsilon_{w^\imath}\hat{w}^\imath_t\right)$$

$$\hat{w}^o_t\left(w^{f*}_t\right) = \left[\phi\Upsilon_a\hat{a}_t + (1-\phi)\Upsilon_{w^\imath}\hat{w}^\imath_t + \phi\Upsilon_{\mathcal{F}}\left(2\hat{\mathcal{F}}_t\left(\hat{w}^{f*}_t - \hat{w}^f_t\right) - \omega^w w^f\frac{\mu}{J^f}\left[\hat{w}^{f*}_t - \hat{w}^f_t\right] - \hat{u}'(c_t)\right)\right]$$

in real terms

$$\hat{w}^o_t\left(w^{f*}_t\right) = \hat{w}^o_t + \varpi_2\left[\hat{w}^f_t - \hat{w}^{f*}_t\right]$$

where $\varpi_2 = \omega^w \mu \phi_w (\phi \Upsilon_{\mathcal{F}} 2)$, $\hat{w}_t^o = \phi \Upsilon_a \hat{a}_t + (1-\phi)\Upsilon_{w^\imath} \hat{w}_t^\imath + \phi \Upsilon_{\mathcal{F}} \left(2\hat{\mathcal{F}}_t - \hat{u}'(c)\right)$. Additionally, the average wage in the formal sector is defined as:

$$\hat{W}_t^f = (1-\omega^w)\hat{W}_t^{f*} + \omega^w \left[\hat{W}_{t-1}^f\right]$$

Combining this expression with the equations that define the evolution of the contract wage, then yields the following second order difference equation for the aggregate wage:

$\hat{W}_t^{f*} = \frac{1}{(1+\Psi)}\hat{w}_t^o + \frac{\Psi}{(1+\Psi)} E_t \left[\hat{W}_{t+1}^{f*}\right]$

$\hat{w}_t^o \left(W_t^{f*}\right) = \hat{w}_t^o + \varpi_2 \left[\hat{W}_t^f - \hat{W}_t^{f*}\right]$

$\hat{W}_t^f = (1-\omega^w)\hat{W}_t^{f*} + \omega^w \left[\hat{W}_{t-1}^f\right]$

$$\hat{W}_t^f = (1-\omega^w)\hat{W}_t^{f*} + \omega^w \left[\hat{W}_{t-1}^f\right]$$

$$\hat{W}_t^f = (1-\omega^w)\left(\frac{1}{(1+\Psi)}\hat{w}_t^o + \frac{\Psi}{(1+\Psi)} E_t \hat{W}_{t+1}^{f*}\right) + \omega^w \left[\hat{W}_{t-1}^f\right]$$

$$(1+\Psi)\hat{W}_t^f = (1-\omega^w)\hat{w}_t^o + (1-\omega^w)\Psi E_t \hat{W}_{t+1}^{f*} + (1+\Psi)\omega^w \left[\hat{W}_{t-1}^f\right]$$

$$\boxed{\hat{W}_t^f = \psi_o \hat{w}_t^o + \psi_1 E_t W_{t+1}^f + \psi_2 \hat{W}_{t-1}^f}$$

where $\varsigma = 1 + \Psi + (\varpi_2 + \Psi)\omega^w$, $\psi_o = \frac{(1-\omega^w)}{\varsigma}$, $\psi_1 = \frac{\Psi}{\varsigma}$, $\psi_2 = \frac{(\varpi_2 + 1 + \Psi)\omega^w}{\varsigma}$. $\psi_o + \psi_1 + \psi_2 = 1$

Appendix B6: Job creation condition and equilibrium condition in the informal sector

In this section, I express the job creation condition in the formal sector and the equilibrium condition in the informal sector as a function of the marginal cost and the formal wage gap.

From the Nash wage bargaining under flexible wages, and the formal job creation condition I have, respectively:

$$w_t^o = \phi\left[a_t + \tfrac{\kappa}{2}\mathcal{F}_t^2 \tfrac{1}{u'(c)} + \theta_t \tfrac{\kappa \mathcal{F}_t}{u'(c)}\right] + [1-\phi]\left[\tfrac{\varphi_t}{u'(c_t)}\right] \tag{68}$$

$$\frac{\kappa \mathcal{F}_{it}}{q(\theta_t)} = \beta E_t \left[u'(c_{t+1})\left(p_{t+1}^f mpl_{t+1}^f - w_{it+1}^f + \frac{\kappa \mathcal{F}_{it+1}^2}{2u'(c_{t+1})}\right) + (1-\rho)\frac{\kappa \mathcal{F}_{it+1}}{q(\theta_{t+1})}\right], \tag{69}$$

Equations (68) and (69) at the steady-state (SS) can be written as follows

$$s_w = \phi\left[\tfrac{l^f}{y}a + s_v + \tfrac{q^w}{\rho}s_v 2\right] + [1-\phi]\left[\ \tfrac{\varphi_t}{u'(c_t)}\tfrac{l^f}{y}\ \right] \tag{70}$$

$$s_v\left[\beta^{-1} - 1 + \frac{\rho}{2}\right] = \frac{\rho}{2}E_t\left[\left(\frac{l^f}{y}a - s_w\right)\right]$$

where $s_w = w\frac{l^f}{y}$, $\rho l^f = q(\theta)v$, $s_v = \frac{hc}{u'(c)c} = \frac{\frac{\kappa}{2}\mathcal{F}_t^2 l_t^f}{u'(c)c}$,

Log-linearizing equations (68) and (69) around the steady-state gives:

$$\frac{2}{\rho}s_v\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_{it}\right) = \beta\left\{\left(\frac{l^f}{y}a\hat{a}_{t+1} - s_w\hat{w}^f_{it+1} + \left(\frac{l^f}{y}a - s_w\right)\hat{u}'(c_{t+1})\right) + \frac{2}{\rho}s_v\hat{\mathcal{F}}_{it+1} + \frac{(1-\rho)\,2}{\rho}s_v\mu\hat{\theta}_{t+1}\right\} \tag{71}$$

$$s_w w^o_t = \phi\frac{l^f}{y}a\hat{a}_t - (1-\phi)\frac{l^f}{y}\frac{\varphi}{u'(c)}\hat{u}'(c_t) + \phi\left(p(\theta)\frac{2}{\rho}s_v + 2s_v\right)\hat{\mathcal{F}}_t - \phi\left(p(\theta)\frac{2}{\rho}s_v + s_v\right)\hat{u}'(c_t) + \phi p(\theta)\frac{2}{\rho}s_v\theta_t \tag{72}$$

Combining both equations in the way that it is possible to express equation (71) in terms of $\left(\hat{w}^f_{t+1} - w^o_{t+1}\right)$ I obtain:

$$\begin{aligned}\tfrac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_{it}\right) &= \left\{\left(\tfrac{l^f}{y}a\hat{a}_{t+1} - s_w\left(\hat{w}^f_{t+1} - w^o_{t+1}\right) + \left(\tfrac{l^f}{y}a - s_w\right)\hat{u}'(c_{t+1})\right) + \tfrac{2}{\rho}s_v\hat{\mathcal{F}}_{it+1} + \tfrac{(1-\rho)2}{\rho}s_v\mu\hat{\theta}_{t+1}\right\} \\ &- \left[\phi\tfrac{l^f}{y}a\hat{a_{t+1}} - (1-\phi)\tfrac{l^f}{y}\tfrac{\varphi}{u'(c)}\hat{u}'(c_{t+1}) + \phi\left(p(\theta)\tfrac{2}{\rho}s_v + 2s_v\right)\hat{\mathcal{F}}_{t+1} - \phi\left(p(\theta)\tfrac{2}{\rho}s_v + s_v\right)\hat{u}'(c_{t+1}) + \phi p(\theta)\tfrac{2}{\rho}s_v\hat{\theta}_{t+1}\right]\end{aligned}$$

reorganizing

$$\begin{aligned}\tfrac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_{it}\right) &= \left\{\left((1-\phi)\tfrac{l^f}{y}a\hat{a}_{t+1} - s_w\left(\hat{w}^f_{t+1} - w^o_{t+1}\right) + \left(\tfrac{l^f}{y}a - s_w\right)\hat{u}'(c_{t+1})\right) + \tfrac{2}{\rho}s_v\hat{\mathcal{F}}_{it+1} + \tfrac{(1-\rho)2}{\rho}s_v\mu\hat{\theta}_{t+1}\right\} \\ &- \left[-\left(s_w - \phi\left[\tfrac{l^f}{y}a + s_v + \tfrac{p(\theta)}{\rho}s_v 2\right]\right)\hat{u}'(c_{t+1}) + \phi\left(p(\theta)\tfrac{2}{\rho}s_v + 2s_v\right)\hat{\mathcal{F}}_{t+1} - \phi\left(p(\theta)\tfrac{2}{\rho}s_v + s_v\right)\hat{u}'(c_{t+1}) + \phi p(\theta)\tfrac{2}{\rho}s_v\hat{\Theta}\right]\end{aligned}$$

then with $s_w - \phi\left[\frac{l^f}{y}a + s_v + \frac{p(\theta)}{\rho}s_v 2\right] = [1-\phi]\left[\ \frac{\varphi_t}{u'(c)}\frac{l^f}{y}\ \right]$

$$\begin{aligned}\tfrac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_{it}\right) &= \left\{\left((1-\phi)\left(\tfrac{l^f}{y}a\hat{a}_{t+1} + \tfrac{l^f}{y}a\hat{u}'(c_{t+1})\right) - s_w\left(\hat{w}^f_{t+1} - w^o_{t+1}\right)\right) + \tfrac{2}{\rho}s_v\hat{\mathcal{F}}_{it+1} + \tfrac{(1-\rho)2}{\rho}s_v\mu\hat{\theta}_{t+1}\right\} \\ &- \left[\phi\left(p(\theta)\tfrac{2}{\rho}s_v + 2s_v\right)\hat{\mathcal{F}}_{t+1} + \phi p(\theta)\tfrac{2}{\rho}s_v\hat{\theta}_{t+1}\right]\end{aligned}$$

$$\begin{aligned}&\tfrac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t + \hat{\mathcal{F}}_{it}\right) = \\ &\left\{\left((1-\phi)\left(\tfrac{l^f}{y}a\hat{a}_{t+1} + \tfrac{l^f}{y}a\hat{u}'(c_{t+1})\right) - s_w\left(\hat{w}^f_{t+1} - w^o_{t+1}\right)\right) + \left(\tfrac{2}{\rho}s_v - \phi\left(p(\theta)\tfrac{2}{\rho}s_v + 2s_v\right)\right)\hat{\mathcal{F}}_{i+1} + \left(\tfrac{(1-\rho)2}{\rho}s_v - p(\theta)\tfrac{2}{\rho}s_v\right)\mu\hat{\theta}_{t+1}\right\}\end{aligned}$$

$$\frac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t+\hat{\mathcal{F}}_t\right)=$$
$$\left\{(1-\phi)\left(p^f\frac{y^f}{y}\left(p^f_{t+1}+mpl^f_{t+1}+\hat{u}'(c_{t+1})\right)+\frac{2}{\rho(1-\phi)}s_v\left(1-\mu\left(p\left(\theta\right)+\rho\right)\right)\hat{\mathcal{F}}_{it+1}+\frac{2}{\rho(1-\phi)}s_v\left(\left(1-\rho\right)-p\left(\theta\right)\right)\mu\hat{\theta}_{t+1}\right)-s_w\left(\hat{w}^f_{t+1}-w^o_{t+1}\right)\right\}$$

I have that prices are equal to the marginal cost, that in perfect competition they should be equal to the marginal income ($\frac{\partial y}{\partial y^f}$)

$$\frac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t+\hat{\mathcal{F}}_t\right)=$$
$$\left\{(1-\phi)\left(\frac{\partial y}{\partial y^f}\frac{y^f}{y}\left(p^f_{t+1}+mpl^f_{t+1}+\hat{u}'(c_{t+1})\right)+\frac{2}{\rho(1-\phi)}s_v\left(1-\mu\left(p\left(\theta\right)+\rho\right)\right)\hat{\mathcal{F}}_{it+1}+\frac{2}{\rho(1-\phi)}s_v\left(\left(1-\rho\right)-p\left(\theta\right)\right)\mu\hat{\theta}_{t+1}\right)-s_w\left(\hat{w}^f_{t+1}-w^o_{t+1}\right)\right\}$$

From the Hosios condition I have $\phi=\mu$, then previous equation becomes

$$\frac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t+\hat{\mathcal{F}}_t\right)=$$
$$\left\{(1-\phi)\left(\frac{\partial y}{\partial y^f}\frac{y^f}{y}\left(p^f_{t+1}+mpl^f_{t+1}+\hat{u}'(c_{t+1})\right)+\frac{2}{\rho(1-\mu)}s_v\left(1-\mu\left(p\left(\theta\right)+\rho\right)\right)\hat{\mathcal{F}}_{i+1}+\frac{2}{\rho(1-\mu)}s_v\left(\left(1-\rho\right)-p\left(\theta\right)\right)\mu\hat{\theta}_{t+1}\right)-s_w\left(\hat{w}^f_{t+1}-w^o_{t+1}\right)\right\}$$

reorganizing

$$\frac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t+\hat{\mathcal{F}}_t\right)=\left\{(1-\phi)\left(\frac{y}{y^f}\right)^{\frac{1}{\gamma}-1}\left(\hat{p}^f_{t+1}-\frac{1}{\gamma}\left(\hat{y}_{t+1}-\hat{y^f}_{t+1}\right)\right)+(1-\phi)\,\Upsilon^f\hat{\Upsilon}^f_t-s_w\left(\hat{w}^f_{t+1}-w^o_{t+1}\right)\right\}$$

$$\frac{2}{\rho\beta}s_v\left(\mu\hat{\theta}_t+\hat{\mathcal{F}}_t\right)=\left\{(1-\phi)\left[\left(\frac{y}{y^f}\right)^{\frac{1}{\gamma}-1}\hat{mc}_{t+1}+\Upsilon^f\hat{\Upsilon}^f_t-\frac{s_w}{(1-\phi)}\left(\hat{w}^f_{t+1}-\hat{w}^o_{t+1}\right)\right]\right\}$$

where $\hat{\Upsilon}^f_t$is the social value of an additional job in the formal sector found in the social planner solution.

Appendix B7: Phillips curve

from the optimal price setting we have

$$p^*_t=\frac{E_t\sum_{\ell=0}^{\infty}\beta^\ell\omega^\ell\left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma}\left(\frac{P_t}{P_{t+\ell}}\right)^{-\Theta}mc_{t+\ell}}{E_t\sum_{\ell=0}^{\infty}\beta^\ell\omega^\ell\left(\frac{c_{t+\ell}}{c_t}\right)^{1-\sigma}\left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta}}$$

$$p^*_t=\frac{\Theta(1-\tau^m)}{(1-\Theta)}\frac{N_t}{D_t}$$

$$\widehat{p^*_t}=\hat{N}_t-\hat{D}_t$$

$$N_t = E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^\ell \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\frac{1}{\sigma}} \left(\frac{P_t}{P_{t+\ell}}\right)^{-\Theta} mc_{t+\ell}$$

$$D_t = E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^\ell \left(\frac{c_{t+\ell}}{c_t}\right)^{1-\frac{1}{\sigma}} \left(\frac{P_t}{P_{t+\ell}}\right)^{1-\Theta}$$

log-linearizing N_t and D_t

$$\hat{D}_t = \omega\beta \left((1-\Theta)\left(\hat{p}_t - \hat{p}_{t+1}\right) + \left(1 - \frac{1}{\sigma}\right)\left(\hat{c}_{t+1} - \hat{c}_t\right) + \hat{D}_{t+1} \right)$$

$$\hat{N}_t = \frac{mc}{N}\hat{mc}_t + \omega\beta \left(-\Theta\left(\hat{p}_t - \hat{p}_{t+1}\right) + \left(1 - \frac{1}{\sigma}\right)\left(\hat{c}_{t+1} - \hat{c}_t\right) + \hat{N}_{t+1} \right)$$

with $\widehat{p}_t^* = \hat{N}_t - \hat{D}_t$ I obtain

$$\widehat{p}_t^* = (1-\omega\beta)\,\hat{mc}_t + \omega\beta\left(\widehat{p}_{t+1}^* + \pi_{t+1}\right) \tag{73}$$

Additionally, the general price index in the formal sector is equal to:

$$P_t = \left(\omega\left(P_{t-1}\right)^{1-\Theta} + (1-\omega)\left(P_t^*\right)^{1-\Theta}\right)^{\frac{1}{1-\Theta}}.$$

$$P_t^{1-\Theta} = \left(\omega\left(P_{t-1}\right)^{1-\Theta} + (1-\omega)\left(P_t^*\right)^{1-\Theta}\right).$$

dividing both sides by $\frac{1}{P_t^{1-\Theta}}$

$$\frac{P_t^{1-\Theta}}{\left(P_{t-1}\right)^{1-\Theta}} = \omega + (1-\omega)\left(\frac{P_t^*}{P_{t-1}}\frac{P_t}{P_t}\right)^{1-\Theta}.$$

log-linearizing around the steady-state

$$\hat{\pi}_t = (1-\omega)\left(\hat{p}_t^* + \hat{\pi}_t\right). \tag{74}$$

replacing (73) into (74) I obtain

$$\hat{\pi} = \kappa\hat{mc}_t + \beta E_t\hat{\pi}_{t+1}$$

with $\kappa = \frac{(1-\omega_p)(1-\omega_p\Gamma)}{\omega_p}$

Appendix B8: wage inflation

By definition, real wage inflation is equal to nominal formal wage inflation minus price inflation,

$$\hat{w}_t^f = \hat{w}_{t-1}^f + \pi_{wt} - \pi_t$$

from equation (39) I have

$$\hat{w}_t^f = \psi_o \hat{w}_t^o + \psi_1 E_t \left(\hat{w}_{t+1}^f + \pi_{t+1} \right) + \psi_2 \left(\hat{w}_{t-1}^f - \pi_t \right)$$

$$\hat{w}_t^f - \hat{w}_{t-1}^f = \psi_o \left(\hat{w}_t^o - \hat{w}_{t-1}^f \right) + \psi_1 E_t \left(\hat{w}_{t+1}^f - \hat{w}_{t-1}^f + \pi_{t+1} \right) + \psi_2 \left(\hat{w}_{t-1}^f - \hat{w}_{t-1}^f - \pi_t \right)$$

$$\pi_{wt} - \pi_t = \psi_o \left(\hat{w}_t^o - \hat{w}_{t-1}^f \right) + \psi_1 E_t \left(\hat{w}_{t+1}^f - \hat{w}_{t-1}^f + \pi_{t+1} \right) + \psi_2 \left(-\pi_t \right)$$

$$\pi_{wt} = \psi_o \left(\hat{w}_t^o - \hat{w}_{t-1}^f \right) + \psi_1 E_t \left(\hat{w}_{t+1}^f - \hat{w}_{t-1}^f + \left(-\hat{w}_{t+1}^f + \hat{w}_t^f + \pi_{wt+1} \right) \right) + (\psi_2 - 1) \left(\hat{w}_t^f - \hat{w}_{t-1}^f - \pi_{wt} \right)$$

$$\psi_2 \pi_{wt} = \psi_o \left(\hat{w}_t^o - \hat{w}_{t-1}^f \right) + \psi_1 E_t \left(\hat{w}_{t+1}^f - \hat{w}_{t-1}^f + \left(-\hat{w}_{t+1}^f + \hat{w}_t^f + \pi_{wt+1} \right) \right) + (\psi_2 - 1) \left(\hat{w}_t^f - \hat{w}_{t-1}^f \right)$$

Since $\psi_o + \psi_1 + \psi_2 = 1$

$$\psi_2 \pi_{wt} = \psi_o \left(\hat{w}_t^o - \hat{w}_t^f \right) + \psi_1 E_t \left(\pi_{wt+1} \right)$$

$$\pi_{wt} = \frac{\psi_o}{\psi_2} \left(\hat{w}_t^o - \hat{w}_t^f \right) + \frac{\psi_1}{\psi_2} E_t \left(\pi_{wt+1} \right),$$

where $\psi_o = \frac{(1-\omega^w)}{\varsigma}$, $\psi_1 = \frac{(\Psi - \varpi_1 \omega^w)}{\varsigma}$, and $\psi_2 = \frac{(\varpi_2 + 1 + \Psi)\omega^w}{\varsigma}$.
Then $\frac{\psi_0}{\psi_2} = \frac{1-\omega^w}{\omega^w \left(1 + \left(1 - \rho + \mu \frac{\rho}{Jf}\right) \omega^w \beta \phi\right)}$, and $\frac{\psi_1}{\psi_2} = \frac{(1-\rho)\phi}{1 + \left(1 - \rho + \mu \frac{\rho}{Jf}\right) \omega^w \beta \phi}$.

Appendix B9: Welfare loss function

The second-order approximation of the welfare criterion

$$U_t = E_t \sum_{t=1}^{\infty} \beta^t \left(\varepsilon_t \right),$$

$$U_t = E_t \sum_{t=1}^{\infty} \beta^t \left(\frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \left(l_t^f + l_t^{\imath} \right) \varphi - \frac{\kappa}{2} \int_0^1 \mathcal{F}_{it}^2 l_{it}^f d_i \right),$$

$$U_t = E_t \sum_{t=1}^{\infty} \beta^t \left(u_t \left(c_t \right) - \left(l_t^f + l_t^{\imath} \right) \varphi - \int_0^1 hc_{it} d_i \right),$$

We can then expand every function in the logarithm of its arguments around their steady-state levels,

$$u_t \left(c_t \right) = u \left(c \right) \left(1 - \sigma^{-1} \right) \left(\hat{c}_t + \frac{1-\sigma^{-1}}{2} \hat{c}_t^2 \right) + t.i.p$$

where $t.i.p$ represents the terms independent of policy. By using $u \left(c \right) \left(1 - \sigma^{-1} \right) = u' \left(c \right) c$ and $c = s_c y$ I obtain

$$u_t \left(c_t \right) = u' \left(c \right) c \left(\hat{c}_t + \frac{1-\sigma^{-1}}{2} \hat{c}_t^2 \right) + t.i.p$$

$$u_t \left(c_t \right) = u' \left(c \right) y \left(s_c \hat{c}_t + \frac{1-\sigma^{-1}}{2} s_c \hat{c}_t^2 \right) + t.i.p$$

Similarly, I do the following approximation

$$\left(l_t^f + l_t^{\imath} \right) \varphi = u' \left(c \right) c \left[\frac{l^f \varphi}{u' \left(c \right) c} \left(\hat{l}_t^f + \frac{1}{2} \left(\hat{l}_t^f \right)^2 \right) + \frac{l^{\imath} \varphi}{u' \left(c \right) c} \left(\hat{l}_t^{\imath} + \frac{1}{2} \left(\hat{l}_t^{\imath} \right)^2 \right) \right] + t.i.p$$

In order to eliminate the linear terms in the previous equation, we need to approximate the aggregate resource constraint.

Individual hiring costs can be written as

$$hc_{it} = \frac{\kappa}{2} \mathcal{F}_{it}^2 l_{it}^f = hc \left[2\hat{\mathcal{F}}_{it} + \hat{l}_{it}^f + \frac{1}{2} \left(2^2 \hat{\mathcal{F}}_{it}^2 + \left(\hat{l}_{it}^f \right)^2 + 2(2) \hat{\mathcal{F}}_{it} \hat{l}_{it}^f \right) \right] + t.i.p$$

employment in the formal sector $l_t^f = \int l_{it}^f d_i$ and the average hiring rate $\bar{\mathcal{F}}_t = \int \mathcal{F}_{it} \frac{l_{it}^f}{l_t^f} di$ can be approximated respectively by

$$\hat{l}_t^f = E_i \hat{l}_{it}^f + \frac{1}{2} Var_i \hat{l}_{it}^f + t.i.p$$

$$\hat{\mathcal{F}}_t = E_i \hat{\mathcal{F}}_{it} + \frac{1}{2} Var \hat{\mathcal{F}}_{it} + E_i \hat{l}_{it}^f \hat{\mathcal{F}}_{it} - \hat{l}_t^f \hat{\mathcal{F}}_t + t.i.p$$

where for any variable e_{it}, $E_i e_{it} \equiv \int_0^1 e_{it} di$ and $Var_i e_{it} \equiv E_i \left(e_{it} - E_i e_{it} \right)^2$denote its cross-sectional average and variance, respectively. I have also used the identity $E_i \left(\hat{l}_{it}^f \right)^2 = Var_i \hat{l}_{it}^f + \left(E_i \hat{l}_{it}^f \right)^2$

and the fact that $\left(\hat{l}_t^f\right)^2=\left(E_i\hat{l}_{it}^f\right)^2$(and similarly for $\hat{\mathcal{F}}_t$). On the other hand, the average hiring rate can also be written as $\mathcal{F}_t=\frac{v_t}{l_t^f}$ which allows me to write $\hat{\mathcal{F}}_t=\hat{v}_t-\hat{l}_t^f$

then

combining the previous three equations, the total hiring costs can be written as follows

$$\int\frac{p_t^f\kappa}{2}\mathcal{F}_{it}^2l_{it}^f=hc\left[2\int\hat{\mathcal{F}}_{it}+\int\hat{l}_{it}^f+\frac{1}{2}\int\left(2^2\hat{\mathcal{F}}_{it}^2+\left(\hat{l}_{it}^f\right)^2+2(2)\hat{\mathcal{F}}_{it}\hat{l}_{it}^f\right)\right]+t.i.p$$

$$=hc\left[2\left(\hat{\mathcal{F}}_t+\tfrac{1}{2}\left(Var\hat{\mathcal{F}}_{it}+\hat{l}_t^f\right)\right)+\tfrac{1}{2}\left(2\hat{\mathcal{F}}_t+\hat{l}_t^f\right)^2\right]+t.i.p$$

with $\hat{\mathcal{F}}_t=\hat{v}_t-\hat{l}_t^f$

$$\int_0^1\frac{\kappa}{2}\mathcal{F}_{it}^2l_{it}^fd_i\quad=u'(c)cs_v\left\{\left(2\hat{v}_t-\hat{l}_t^f\right)+\tfrac{1}{2}\left[\left(2\hat{v}_t-\hat{l}_t^f\right)^2+2Var\hat{\mathcal{F}}_{it}\right]\right\}+t.i.p \tag{75}$$

$$\int_0^1\frac{\kappa}{2}\mathcal{F}_{it}^2l_{it}^fd_i\quad=u'(c)cs_v\left\{\left(2\hat{v}_t-\hat{l}_t^f\right)+\tfrac{1}{2}\left[\left(2\hat{v}_t-\hat{l}_t^f\right)^2+2Var\hat{\mathcal{F}}_{it}\right]\right\}+t.i.p$$

where $s_v=\frac{hc}{u'(c)c}=\frac{\frac{\kappa}{2}\mathcal{F}_t^2l_t^f}{u'(c)c}$ is the vacancy posting cost in consumption units as a fraction of GDP

therefore

$$U_t=u'(c)\,c\left(\hat{c}_t+\tfrac{1-\sigma^{-1}}{2}\hat{c}_t^2-l^f\tfrac{\varphi}{u'(c)c}\left(\hat{l}_t^f+\tfrac{1}{2}\left(\hat{l}_t^f\right)^2\right)-l^\imath\tfrac{\varphi}{u'(c)c}\left(\hat{l}_t^\imath+\tfrac{1}{2}\left(\hat{l}_t^\imath\right)^2\right)+t.i.p\right)$$
$$-u'(c)cs_v\left\{\left(2\hat{v}_t-\hat{l}_t^f\right)+\tfrac{1}{2}\left[\left(2\hat{v}_t-\hat{l}_t^f\right)^2+2Var\hat{\mathcal{F}}_{it}\right]\right\}+t.i.p$$

we have $y_t=\triangle_tc_t$, then $\hat{y}_t=\triangle_t+\hat{c}_t$ and from the equilibrium in the intermediate good market

$\left(\hat{l}_t^\imath+t.p.i\right)=\hat{y}_t^\imath$

$\left(\hat{l}_t^f\right)+t.p.i=\hat{y}_t^f$

$\hat{y}_t=\Psi_{yf}\widehat{y_t^f}+\Psi_{yi}\hat{y}_t^\imath=\triangle_t+\hat{c}_t$

$\hat{y}_t=\Psi_{yf}\widehat{l_t^f}+\Psi_{yi}\hat{l}_t^\imath+t.i.p=\triangle_t+\hat{c}_t$

$$U_t=u'(c)\,c\left(\left(\Psi_{yf}-l^f\tfrac{\varphi}{u'(c)c}\right)\widehat{l_t^f}+\left(\Psi_{yi}-l^\imath\tfrac{\varphi}{u'(c)c}\right)\hat{l}_t^\imath-\triangle_t+\tfrac{1-\sigma^{-1}}{2}\hat{y}_t^2-l^f\tfrac{\varphi}{u'(c)c}\tfrac{1}{2}\left(\hat{l}_t^f\right)^2-l^\imath\tfrac{\varphi}{u'(c)c}\tfrac{1}{2}\left(\hat{l}_t^\imath\right)^2\right)$$
$$-u'(c)cs_v\left\{\left(2\hat{v}_t-\hat{l}_t^f\right)+\tfrac{1}{2}\left[\left(2\hat{v}_t-\hat{l}_t^f\right)^2+2Var\hat{\mathcal{F}}_{it}\right]\right\}+t.i.p$$

The Beveridge Curve and law of motion of the employment in the informal sector

In order to eliminate the linear terms in the previous equation, I perform the following second order approximation of the law of motion of employment in the formal and in the informal sector

$$l_{t+1}^{f} = (1-\rho)\, l_t^{f} + \mathbb{N}\, (l_t^{u})^{\mu}\, (\bar{v}_t)^{1-\mu}$$

then

$$\hat{l}_{t+1}^{f} + \frac{1}{2}\left(\hat{l}_{t+1}^{f}\right)^2 = (1-\rho)\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) + \rho\left[\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t + \frac{1}{2}\left(\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t\right)^2\right] + \mathcal{O}^3$$

$$l^{u} = 1 - l^{f} - l^{\imath}$$

$$l^{u}\left(\hat{l}_t^{u} + \frac{1}{2}\left(\hat{l}_t^{u}\right)^2\right) = -l^{f}\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) - l^{\imath}\left(\hat{l}_t^{\imath} + \frac{1}{2}\left(\hat{l}_t^{\imath}\right)^2\right) + \mathcal{O}^3$$

$$\hat{l}_t^{u} = -\frac{l^{f}}{l^{u}}\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) - \frac{l^{\imath}}{l^{u}}\left(\hat{l}_t^{\imath} + \frac{1}{2}\left(\hat{l}_t^{\imath}\right)^2\right) - \frac{1}{2}\left(\hat{l}_t^{u}\right)^2 + \mathcal{O}^3$$

or

$$\left(\hat{l}_t^{\imath}\right) = -\frac{l^{u}}{l^{\imath}}\left(\hat{l}_t^{u} + \frac{1}{2}\left(\hat{l}_t^{u}\right)^2\right) - \frac{l^{f}}{l^{\imath}}\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) - \frac{1}{2}\left(\hat{l}_t^{\imath}\right)^2 + \mathcal{O}^3$$

replacing in

$$\hat{l}_{t+1}^{f} + \frac{1}{2}\left(\hat{l}_{t+1}^{f}\right)^2 = (1-\rho)\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) + \rho\left[\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t + \frac{1}{2}\left(\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t\right)^2\right] + \mathcal{O}^3$$

multiplying by β^t and iterating across t

$$\left(\beta^{-1} - (1-\rho)\right)\sum_{t=0}^{\infty}\beta^t\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) = \sum_{t=0}^{\infty}\beta^t\rho\left[\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t + \frac{1}{2}\left(\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t\right)^2\right] + \mathcal{O}^3 \tag{76}$$

Reorganizing

$$\left(\beta^{-1} - (1-\rho)\right)\sum_{t=0}^{\infty}\beta^t\left(\hat{l}_t^{f} + \frac{1}{2}\left(\hat{l}_t^{f}\right)^2\right) = \sum_{t=0}^{\infty}\beta^t\rho\left[\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t + \frac{1}{2}\left(\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t\right)^2\right] + t.i.p$$

$$\sum_{t=0}^{\infty}\beta^t\left[\left(\beta^{-1} - (1-\rho)\right)\hat{l}_t^{f} - \rho\left((1-\mu)\,\hat{v}_t + \mu\hat{l}_t^{u}\right)\right] = \sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left[\rho\left(\mu\hat{l}_t^{u} + (1-\mu)\,\hat{v}_t\right)^2 - \left(\beta^{-1} - (1-\rho)\right)\left(\hat{l}_t^{f}\right)^2\right] + t.i.p$$

from the efficient job creation condition in the steady-state I have

$$\left(\beta^{-1} - (1-\rho)\right) = (1-\mu)\,\frac{\rho}{2}\left[s_v^{-1}\left(\left(\frac{y}{y^{f}}\right)^{\frac{1-\gamma}{\gamma}} - \left(\frac{y}{y^{\imath}}\right)^{\frac{1-\gamma}{\gamma}}\frac{l^{f}}{l^{\imath}}\right) + 1\right]$$

$$(1-a)\left(\frac{y}{y^{\imath}}\right)^{\frac{1-\gamma}{\gamma}}\frac{l_t^u}{l^{\imath}}=2s_v\frac{\mu}{(1-\mu)}$$

combining the two following equations

$$\left(\beta^{-1}-(1-\rho)\right)=(1-\mu)\frac{\rho}{2}\left[s_v^{-1}\left(\left(\frac{y}{y^f}\right)^{\frac{1-\gamma}{\gamma}}-\left(\frac{y}{y^{\imath}}\right)^{\frac{1-\gamma}{\gamma}}\frac{l^f}{l^{\imath}}\right)+1\right]$$

$$\sum_{t=0}^{\infty}\beta^t\left[\left(\beta^{-1}-(1-\rho)\right)\hat{l}_t^f-\rho\left((1-\mu)\,\hat{v}_t+\mu\hat{l}_t^u\right)\right]=\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left[\rho\left(\mu\hat{l}_t^u+(1-\mu)\,\hat{v}_t\right)^2-\left(\beta^{-1}-(1-\rho)\right)\left(\hat{l}_t^f\right)^2\right]+t.i.p$$

I obtain

$$\begin{gathered}\textstyle\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left\{\left[s_v^{-1}\left(\left(\frac{y}{y^f}\right)^{\frac{1-\gamma}{\gamma}}-\left(\frac{y}{y^{\imath}}\right)^{\frac{1-\gamma}{\gamma}}\frac{l^f}{l^{\imath}}\right)+1\right]\hat{l}_t^f-2\left(\hat{v}_t+\frac{\mu}{(1-\mu)}\hat{l}_t^u\right)\right\}=\\ \textstyle\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left\{\frac{1}{(1-\mu)}\left(\mu\hat{l}_t^u+(1-\mu)\,\hat{v}_t\right)^2-\frac{1}{2}\left[s_v^{-1}\left(\left(\frac{y}{y^f}\right)^{\frac{1-\gamma}{\gamma}}-\left(\frac{y}{y^{\imath}}\right)^{\frac{1-\gamma}{\gamma}}\frac{l^f}{l^{\imath}}\right)+1\right]\left(\hat{l}_t^f\right)^2\right\}+t.i.p\end{gathered}$$

and combining with

$$\begin{gathered}\textstyle\sum_{t=1}^{\infty}\beta^tU_t=u'(c)\,cs_v\left\{-\triangle_t\right\}\\ \textstyle-\sum_{t=1}^{\infty}\beta^tu'(c)\,c\frac{1}{2}\left[-\left(1-\sigma^{-1}\right)\hat{y}_t^2+\left(\Psi_{yf}\right)\left(\hat{l}_t^f\right)^2+\left(\Psi_{yi}\right)\left(\hat{l}_t^{\imath}\right)^2\right]\\ \textstyle-\sum_{t=1}^{\infty}\beta^tu'(c)cs_v\left\{\left[\mu\left(\theta_t\right)^2+\left(\mathcal{F}_t\right)^2+Var\hat{\mathcal{F}}_{it}\right]\right\}+t.i.p\end{gathered}$$

then, reorganizing I have

$$\textstyle\sum_{t=1}^{\infty}\beta^tU_t=\sum_{t=1}^{\infty}\beta^t\frac{u'(c)c}{2}\left\{\begin{array}{c}-2\hat{\triangle}_t-\left(\sigma^{-1}-1\right)\hat{y}_t^2-2s_v\left[\mu\hat{\theta}_t^2+\mathcal{F}^2+Var\hat{\mathcal{F}}_{it}\right]\\ -\left(\Psi_{yf}\right)\left(\hat{l}_t^f\right)^2-\left(\Psi_{yi}\right)\left(\hat{l}_t^{\imath}\right)^2+t.i.p\end{array}\right\}$$

Price dispersion and inflation

The second order Taylor expansion of $\triangle_t=\int_0^1\left(\frac{p_{jt}}{p_t}\right)^{-\Theta}dj$ writes:

$$\hat{\triangle}_t+\frac{1}{2}\hat{\triangle}_t^2=-\Theta\left(E_j\hat{p}_{jt}-\frac{\Theta}{2}E_j\left(\hat{p}_{jt}\right)^2\right)+\mathcal{O}^3$$

where

$\hat{p}_{jt}=log\left(\frac{p_{jt}}{p_j}\right)$ and we have $\triangle=1.\hat{\triangle}_t$ is proportional to the cross-sectional variance of relative prices. Therefore, $\hat{\triangle}_t\simeq\frac{\Theta}{2}var_i\left\{p_t(i)\right\}$

In Woodford (2003, chapter 6) is proved that

$$\sum_{t=0}^{\infty}\beta^tvar_i\left\{p_t(i)\right\}=\frac{\omega}{(1-\beta\omega)(1-\omega)}\sum_{t=0}^{\infty}\beta^t\pi_t^2$$

then

$$\sum_{t=0}^{\infty} \beta^t \hat{\triangle}_t \simeq \sum_{t=0}^{\infty} \beta^t \frac{\Theta}{2} var_i \left\{ p_t(i) \right\}$$

$$\sum_{t=0}^{\infty} \beta^t \hat{\triangle}_t \simeq \sum_{t=0}^{\infty} \beta^t \frac{\Theta}{2} \frac{\omega}{(1-\beta\omega)(1-\omega)} \pi_t^2$$

with $\Upsilon = \frac{(1-\beta\omega)(1-\omega)}{\omega}$ we ca express U_t as follows

$$\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t \frac{u'(c)c}{2} \left\{ \begin{array}{c} -\frac{\Theta}{\Upsilon}\pi_t^2 - \left(\sigma^{-1} - 1\right) \hat{y}_t^2 - 2s_v \left[\mu \hat{\theta}_t^2 + \mathcal{F}^2 + Var\hat{\mathcal{F}}_{it} \right] \\ -\left(\Psi_{yf}\right) \left(\hat{l}_t^f\right)^2 - \left(\Psi_{yi}\right) \left(\hat{l}_t^i\right)^2 + t.i.p \end{array} \right\} \tag{77}$$

Wage inflation and hiring rate dispersion

Analogously, the cross-sectional variance of nominal wages can be approximated by

$$var_i log\left(w_{it}^f\right) = \omega^w var_i log\left(w_{it-1}^f\right) + \frac{\omega^w}{1-\omega^w}\pi_{wt}^2 \tag{78}$$

Multiplying (78) by β^t integrating forward and using the fact that $var_i log\left(w_{it-1}\right)$ is independent of policy as of time 0, I obtain

$$\sum_{t=0}^{\infty} \beta^t var_i log\left(w_{it}^f\right) = \frac{\omega^w}{(1-\omega^w)(1-\beta\omega^w)} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2 + t.i.p$$

By using Lemma 1 in Thomas (2008) I found that

$$Var_i \hat{\mathcal{F}}_{it} = \hbar^2 var_i log(w_{it}^f)$$

where $\hbar = \frac{\beta\omega^w s_w}{(1-\beta\omega^w)\frac{2}{\rho}s_v}$, $s_w = \frac{l^f w}{y}$ is the steady state formal labor income share, s_v is the steady state ratio of vacancy posting cost (in consumption units) to output

then it is possible to write

$$\sum_{t=0}^{\infty} \beta^t Var_i \hat{\mathcal{F}}_{it} = \frac{\hbar^2 \omega^w}{(1-\omega^w)(1-\beta\omega^w)} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2 + t.i.p \tag{79}$$

$$\sum_{t=0}^{\infty} \beta^t Var_i \hat{\mathcal{F}}_{it} = \frac{\hbar^2}{\Upsilon_w} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2 + t.i.p \tag{80}$$

with $\Upsilon_w = \frac{(1-\omega^w)(1-\beta\omega^w)}{\omega^w}$

finally inserting (79) into (77)

$$\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t \frac{u'(c)c}{2} \left\{ \begin{array}{c} -\frac{\Theta}{\Upsilon}\pi_t^2 - \left(\sigma^{-1} - 1\right) \hat{y}_t^2 - 2s_v \left[\mu\hat{\theta}_t^2 + \mathcal{F}^2 + \frac{\hbar^2}{\Upsilon_w}\pi_{wt}^2\right] \\ - \left(\Psi_{yf}\right) \left(\hat{l}_t^f\right)^2 \text{-} \left(\Psi_{yi}\right) \left(\hat{l}_t^\imath\right)^2 + t.i.p \end{array} \right\}$$

$$\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t \frac{u'(c)c}{2} \left\{ \begin{array}{c} -\Psi_\pi \pi_t^2 - \Psi_{\pi w}\pi_{wt}^2 - \left(\sigma^{-1} - 1\right) \hat{y}_t^2 - 2s_v \left[\mu\hat{\theta}_t^2 + \mathcal{F}^2\right] \\ - \left(\Psi_{yf}\right) \left(\hat{l}_t^f\right)^2 \text{-} \left(\Psi_{yi}\right) \left(\hat{l}_t^\imath\right)^2 + t.i.p \end{array} \right\}$$

$$\sum_{t=1}^{\infty} \beta^t U_t = -\sum_{t=1}^{\infty} \beta^t \frac{u'(c)c}{2} \left\{ \begin{array}{c} \Psi_\pi \pi_t^2 + \Psi_{\pi w}\pi_{wt}^2 + \left(\sigma^{-1} - 1\right) \hat{y}_t^2 + 2s_v \left[\mu\hat{\theta}_t^2 + \mathcal{F}^2\right] \\ - \left(\Psi_{yf}\right) \left(\hat{l}_t^f\right)^2 \text{-} \left(\Psi_{yi}\right) \left(\hat{l}_t^\imath\right)^2 + t.i.p \end{array} \right\}$$

where $\Psi_\pi = \frac{\Theta}{\Upsilon}$ and $\Psi_{\pi w} = s_v 2\frac{\hbar^2}{\Upsilon_w}$, $\Psi_{yf} = \left(\frac{y}{y^f}\right)^{\frac{1-\gamma}{\gamma}}$, $\Psi_{yi} = \left(\frac{y^\imath}{y}\right)^{\frac{\gamma-1}{\gamma}}$

$$\sum_{t=0}^{\infty} \beta^t U_t = -\sum_{t=0}^{\infty} \beta^t \frac{u'(c)c}{2} \left\{\Psi_\pi \pi_t^2 + \Psi_{\pi w}\pi_{wt}^2 + \left(\sigma^{-1} - 1\right) \hat{y}_t^2 + 2s_v \left[\mu\hat{\theta}_t^2 + \mathcal{F}^2\right] + \left(\Psi_{yf}\right) \left(\hat{l}_t^f\right)^2 + \left(\Psi_{yi}\right) \left(\hat{l}_t^\imath\right)^2\right\} + t.i.p$$

$$\sum_{t=0}^{\infty} \beta^t U_t = -\sum_{t=0}^{\infty} \beta^t \frac{u'(c)c}{2} L_t + t.i.p$$

with

$$L_t = \Psi_\pi \pi_t^2 + \Psi_{\pi w}\pi_{wt}^2 + \mathcal{L}_t^{l,h}$$

$$\mathcal{L}_t^{l,h} = \left(\sigma^{-1} - 1\right) \hat{y}_t^2 + 2s_v \left[\mu\hat{\theta}_t^2 + \mathcal{F}^2\right] + \Psi_{yf} \left(\hat{l}_t^f\right)^2 + \Psi_{yi} \left(\hat{l}_t^\imath\right)^2.$$

6.1 References

Alberola, E., & Urrutia, C. (2020). Does informality facilitate inflation stability? *Journal of Development Economics, 146(C).*

Albrecht, J. Navarro, L., & Vroman, S. (2009). The Effects of Labor Market Reforms in an Economy with an Informal Sector. *The Economic Journal*, 119, 1105–1129.

Badaoui, E., Strobl, E., & Walsh, F. (2006). An Equilibrium Search Model of the Informal Sector. Working Paper Series WP06/29, University College Dublin. School of Economics.

Barro, Robert. J.(1977). Long-term contracting, sticky prices, and monetary policy. *Journal of Monetary Economics,* 3(3), 305–316.

Batini, N., Levine, P., Lotti, E., & Yang, B. (2011). Monetary and Fiscal Policy in the Presence of Informal Labour Markets. Working Papers 11/97, National Institute of Public Finance and Policy.

Blades, D., Ferreira, F., & Lugo, M. A. (2011). The informal economy in developing countries: an Introduction. *Review of Income and Wealth*, Series 57, Special Issue, May.

Blanchard, O., & Galí, J. (2010). Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment. *American Economic Journal: Macroeconomics,* 2(2), 1-30.

Boeri, T. & Garibaldi, P. (2005). Shadow Sorting. in Pissarides C. and J. Frenkel NBER Macroeconomics Annual MIT Press.

Brueckner, J., & Zenou, Y. (1999). Harris-Todaro models with a land market. *Regional Science and Urban Economics*, 29, 317–339.

Calvo, Guillermo. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3), 383-398.

Cardozo, Miguel (2019). Determinantes de la duracion de las vacantes en el mercado laboral colombiano. Monografia, Pontificia Universidad Javeriana.

Castillo, P., & Montoro, C. (2010). Monetary Policy in the Presence of Informal Labour Markets. Working paper, Banco Central de Reserva del Peru.

Fernández, A., & Meza, F., (2015). Informal employment and business cycles in emerging economies: the case of Mexico. *Review of Economic Dynamics,* 18 (2), 381–405.

Fernandez, Cristina (2018). Informalidad empresarial en Colombia. Working Papers Series, No. 017196, Fedesarrollo.

Fugazza, M., & Jacques, J-F. (2003). Labor market Institutions, taxation and the underground economy. J*ournal of Public Economics,* 88, 395–418.

Gertler, M., & Trigari, A. (2009). Unemployment Fluctuations with Staggered Nash Wage Bargaining. *Journal of Political Economy,* 117(1), 38-86.

Hall, Robert E. (2005). Employment fluctuations with equilibrium wage stickiness. *American Economics Review,* 95 (1), 50–65.

Harris, J., & Todaro, M. (1970). Migration, Unemployment, and Development: A two sector Analysis. *American Economic Review,* 40, 126–142.

Hosios, Arthur J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies,* 57 (2), 279–298.

ILO (2018). Women and men in the informal economy: a statistical picture (third edition) / International Labour Office – Geneva: ILO, 2018.

Lewis, A. (1954). Economic Development with Unlimited Supplies of Labour. Manchester School, 22, 139–191.

Leyva, G., & Urrutia, C. (2018). Informality, Labor Regulation, and the Business Cycle. Banco de México, Documento de Investigación, 2018-19.

Restrepo-Echavarria, Paulina. (2014). Macroeconomic Volatility: The Role of the Informal Economy. *European Economic Review*, 70(C), 454–469.

Rogerson, Richard (1988) Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics*, 21, 3-16.

Satchi, M. & Temple, J. (2009). Labor Markets and Productivity in Developing Countries. *Review of Economic Dynamics,* 26, 333–345.

Thomas, Carlos (2008). Search and matching frictions and optimal monetary policy. *Journal of Monetary Economics*, 55(5), 936-956.

Ulyssea, Gabriel. (2018). Firms, Informality, and Development: Theory and Evidence from Brazil. *The American Economic Review,* 108(8), 2015-2047.

Woodford, Michael (2003). Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton, NJ.

World Bank. (2019). Global Economic Prospects, January 2019: Darkening Skies. Washington, DC: World Bank. doi: 10.1596/978-1-4648-1343-6. License: Creative Commons Attribution CC BY 3.0 IGO.

Zenou, Yves. (2008). Job Search and Mobility in Developing Countries: Theory and Policy Implications. *Journal of Development Economics,* 86, 336–355.