Tax Evasion, Informality, and Optimal Monetary Policy*

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July 2021

Abstract

Our paper aims to unveil how much monetary policy shall deviate from the flexible-price allocation in an economy with a large informal sector. First of all, the presence of variable taxes in the formal sector generates an inflation bias under a discretionary policy, which increases with the size of the informal sector. Secondly, we find that only the formal sector is responsible for the cost-push shocks that are amplified in a more informal economy. The trade-off between inflation and the formal output gap is then dependent on the elasticity of the former variable with respect to the formal output gap. However, the optimal management of inflation also depends on the elasticity of the informal output gap with respect to the formal output gap. As this elasticity is decreasing with the size of the informal sector, whether inflation volatility (in terms of the aggregate output gap) is lower or higher in a more informal economy is ambiguous. By simulation, we show that economies with a larger informal sector should stabilize less inflation relative to the total output gap.

Keywords: Informality, optimal monetary policy, New-Keynesian macroeconomics, tax distortion.

JEL classification: E26, E52, E12, H21

*An early version of this paper circulate before with the title "Optimal monetary policy with informality: a first pass"

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1 Introduction

Surprisingly enough, a small number of papers have been devoted to the analysis of the monetary policy when the economy displays an informal sector, where value-added activities avoid taxation. A lot of countries share these features, especially emerging countries where the rule of law and tax compliance are not well established[1]. Generally, informal labor markets are the result of agents who want to avoid taxation and regulation, despite the protection and advantages that the state can provide in the formal sector; it is also the outcome of agents who cannot find a job in the formal sector and depend on informal jobs as a means of subsistence.

Given the importance of the labor market structure in determining output, inflation, and the response of the economy to aggregate shocks, it is of great importance to analyze the implications of informality for monetary policy in developing countries. What is then the effect of the informality scale on the optimal trade-off between inflation and output or the inflation bias? From the seminal work of Clarida et al. (1999), monetary policy has been built on very firm theoretical foundations. The New Keynesian (NK) framework offers clear guidelines for central banks of developed countries. It remains to propose a canonical model encompassing what is the major issue for emerging countries: the existence of a large informal sector. Our paper aims at deriving the first principles of monetary policy according to the relative size of the informal sector. We propose a canonical model in the NK framework in order to derive analytically these principles. We choose to focus on tax avoidance as the key feature of the informal sector: our model is an NK two-sector economy with taxation only in the formal sector. In this simple model, it is possible to derive the optimal policy recommendations from an approximated quadratic welfare function, and then to characterize the role of the informality size for monetary policy analytically.

We show that the presence of distortive taxes generates an inflation bias under the discretionary policy as a result of the central bank's incentive to boost production above its natural level. This inflationary bias increases with the size of the informal sector, even though only the formal sector displays a distorted steady state. Additionally, we

[1] Labor markets in developing countries are particularly affected by the existence of a large informal sector. According to the ILO (2018), informal employment accounts for more than half of non-agricultural employment in most developing countries: around 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States, 50 percent in Latin America, and 30 percent in Europe and Central Asia. In the case of developed countries, only 17 percent of the urban labor force is employed in informal activities.

find that only the formal sector due to tax distortion fluctuations is responsible for cost-push shocks: in this sector, the gap between the natural rate and the first best allocation varies due to fluctuations in tax distortions. On the other hand, cost-push shocks are amplified in a more informal economy. The trade-off between inflation and the formal output gap is then dependent on the elasticity of the former variable with respect to the latter one (the inverse of the sacrifice ratio in terms of formal output). Unambiguously, this elasticity is lower in a more informal sector, which would lead to higher (relative) volatility of inflation in such an economy. However, this is only one dimension of the optimal management of inflation, as the formal output gap is only one dimension of the policy-relevant aggregate output gap, which must also take into account the elasticity of the informal output gap with respect to the formal output gap (the sectoral integration). As this elasticity is decreasing with the size of the informal sector, whether inflation volatility (in terms of the aggregate output gap) is lower or higher in a more informal economy is ambiguous. By simulation, we show that economies with a larger informal sector should stabilize less inflation relative to the total output gap.

Our paper is the first one to investigate the implication of informality for monetary policy in the standard NK framework. Castillo and Montoro (2010) and Batini et al. (2011) propose too complex theoretical frameworks to derive analytical results. Our simple model can deliver clear insights about inflation dynamics: emerging economies with a large informal sector should display higher mean and lower inflation volatility (relative to formal and aggregate output gaps).

A key assumption in our model is the absence of public debt, and then the absence of tax smoothing allowed by debt management over the business cycle. It is certainly an extremely simplifying assumption, but it allows us to unveil some basic properties. It also reflects the fact that in emerging countries the public debt management is more constrained than in developed countries. Overall, our framework certainly overemphasizes tax variations over the business cycles, but we do believe that it delivers the key differences of the optimal monetary policy across developed and emerging countries[2].

The rest of the chapter is organized as follows. In Section 2 we develop a NK model with two sectors, and endogenous labor income taxes in the formal sector. In Section

[2]According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 to 20 percent of GDP, while the average for high-income countries is more like 40 percent. This difference is not necessarily explained by a choice of low tax rates but by the challenges associated with tax collection: these include informality and misreporting (Besley and Persson 2009).

3, we show how the presence of varying taxes generates a trade-off between stabilizing inflation and stabilizing the policy-relevant output gap. In Section 4 we characterize optimal monetary policy. In section 5 we realize a quantitative analysis of the model. Section 6 concludes.

2 A New Keynesian model with informality

We propose a New-Keynesian closed economy framework with a formal sector (F) and an informal one (I). Only workers in the formal sector have to pay a wage income tax, which is a source of distortions for working hours. We consider an economy populated by infinitely-lived households whose utility depends on leisure and the consumption of market goods produced by a continuum of monopolistically competitive firms. Formal and informal firms hire labor from the households to produce a wholesale good which is sold in a competitive market. Retail firms use the wholesale good as an input and transform it into differentiated final goods.

2.1 Households

An exogenous fraction N_S of individuals works in sector, S, $S \in (F,\ I)$, and maximize the following expected discounted utility function:

$$U_{S,t} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{S,t}^{1-\sigma} - 1}{1-\sigma} - \frac{{h_{S,t}}^{1+\eta}}{1+\eta} \right] \tag{1}$$

Households in the formal sector have to pay a labor income tax τ_t^w. They maximize their utility subject to the following budget constraint:

$$c_{F,t} + b_{F,t} \leq (1 - \tau_t^w)\, w_{F,t} N_F h_{F,t} + \frac{(1 + i_{t-1})}{1 + \pi_t} b_{F,t-1} + T_F \tag{2}$$

On the other side, households in the informal sector consume their current income and do not pay taxes. Their budget constraint writes:

$$c_{I,t} \leq N_I w_{I,t} h_{F,t} \tag{3}$$

where w_S is the real wage in sector S, c_S is the aggregate CES basket of i goods consumed by the S sector, b_F is a one-period bond and h_S is the working hours supplied by individuals in sector S, and $\pi_t = \left(\frac{P_t}{P_{t-1}} - 1\right)$ is the inflation rate between $t-1$ and t (where P_t is the level of prices in t). $\beta = \frac{1}{1+\rho}$, with $\rho > 0$, is the subjective discount rate. σ and η are positive parameters which define the curvature of leisure and consumption preferences. τ^w is the tax rate paid by workers in the formal sector. The choice of working hours in the formal sector is then distorted. T_F is a lump-sum transfer to households in the formal sector.

The first-order conditions of the intertemporal program for households working in the formal and the informal sectors are given by:

$$\left(\frac{c_{F,t+1}}{c_{F,t}}\right)^{\sigma} = \beta E_t\left(\frac{1+i_t}{1+\pi_{t+1}}\right) \tag{4}$$

$$h_{F,t}^{\eta} = (c_{F,t})^{-\sigma}\left(1-\tau_t^w\right) w_{F,t} \tag{5}$$

$$h_{I,t}^{\eta} = (c_{I,t})^{-\sigma} w_{I,t} \tag{6}$$

Each period, households choose optimally the quantity of each variety j:

$$c_{S,jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} c_{S,t}, \tag{7}$$

where θ measures the elasticity of substitution between varieties.

2.2 Firms

Wholesale firms

Formal and informal firms in the wholesale sector hire $N_F h_{F,t}$ and $N_I h_{I,t}$ labor hours from households in order to produce $Y_{F,t}$ and $Y_{I,t}$ units of the intermediate good, using the following technology:

$$Y_{S,t} = A_{S,t} N_S h_{S,t} \tag{8}$$

where $A_{S,t}$ is the stochastic sectoral productivity common to all firms in sector S. We assume that $ln(A_{S,t})$ follows a first-order stationary auto-regressive process with auto-regressive coefficient ρ_S.

In a competitive environment, the maximization of profits implies that the wholesale real price P^Y equals the real marginal cost $\phi_{S,t}$ in each sector:

$$P_t^Y = \frac{w_{S,t}}{A_{S,t}} \equiv \phi_{S,t} \quad \forall S = F, I \tag{9}$$

Retail firms

Retail firms owned by households working in the formal sector purchase wholesale output at price P^Y and transform it without labor or capital into differentiated final goods j. Following Calvo (1983), each monopolistic retailer is assumed to reset its price with probability $(1-\omega)$ in any given period, independent of the time elapsed since their last adjustment. A firm that can adjust its price in period t chooses P_t^* to maximize its intertemporal flows of profits:

$$\underset{P_t^*}{max} \quad E_t \sum_{j=0}^{\infty} \mathit{\Gamma}_{t,t+j} \omega^j \left[P_t^* Y_{t+j/t} - P_{t+j}(1-\tau^m) P_{t+j}^Y Y_{t+j/t} \right]$$

subject to the sequence of demand constraints:

$$Y_{t+j/t} = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\theta} Y_{t+j}$$

where $\mathit{\Gamma}_{t,t+j} = \beta^j \left(\frac{c_{F,t+j}}{c_{F,t}} \right)^{-\sigma} \left(\frac{P_t}{P_{t+j}} \right)$ is the stochastic discount factor for nominal payoffs, and $Y_{t+j/t}$ denotes the output in period $t+j$ for a firm whose last reset of prices was in period t. In order to focus on the distortions imposed by taxation in the formal sector, a subsidy $\tau^m = \frac{1}{\theta}$ financed by lump-sum taxes is assumed to offset the monopolistic distortion at the steady state.

2.3 Government

The government runs a balanced budget. The distortive taxes to formal workers are used to finance constant public transfers to households in the formal sector (T_F). Therefore, government´s budget constraint regarding these transfers writes as follows[3]:

$$\tau_t^w N_F w_{F,t} h_{F,t} = T_F \tag{10}$$

The tax rate τ_t^w varies over the business cycle in order to balance the fluctuating tax base.

3 Phillips, Sectoral Integration and IS curves

In what follows, the variables with hat represent the log-deviation of the variable from their steady state value.

Sectoral integration curve

Using Equations (9), (5) and (6), we derive, after log-linearizing around the steady state, the following expressions for the marginal cost in the informal and formal sectors (See Appendix A1 for derivation):

$$\hat{\phi}_{I,t} = (\eta + \sigma)\,\mathbb{X}_{I,t} \tag{11}$$

and

$$\hat{\phi}_{F,t} = (\eta + \sigma)\,\mathbb{X}_{F,t} + \frac{\tau^w}{1-\tau^w}\hat{\tau}_t^w \tag{12}$$

where $\mathbb{X}_{S,t} = \hat{Y}_{S,t} - \hat{Y}_{S,t}^e$ is the *welfare-relevant* output gap in sector S, i.e. the deviation between the actual output $\hat{Y}_{S,t}$ and its efficient level $\hat{Y}_{S,t}^e$. The marginal cost in the informal sector is proportional to the (welfare-relevant) output gap, whereas it also depends on tax variations in the formal sector.

From Equation (9), $\hat{\phi}_{F,t} = \hat{\phi}_{I,t}$. This sectoral integration condition implies a relationship between the sectoral output gaps:

$$(\eta + \sigma)\,(\mathbb{X}_{F,t} - \mathbb{X}_{I,t}) = -\frac{\tau^w}{1-\tau^w}\hat{\tau}_t^w$$

[3]On the other hand, the sum of all subsidies given to retailers at the steady state are fully financed by lump-sum taxes.

A change in labor tax impacts the marginal cost in the formal sector and implies working hours differential across sectors over the business cycle. Cyclical variations in the tax disconnect the formal output gap from the informal one, especially in an economy with a large informal sector. In an otherwise homogeneous economy across sectors, the two sectoral output gaps would perfectly co-move. Taking into account the expression of the tax variation:

$$\hat{\tau}_t^w = -\hat{\phi}_{F,t} - \hat{A}_{F,t} - \hat{h}_{F,t} \tag{13}$$

we get:

$$\mathbb{X}_{I,t} = \Omega_x \mathbb{X}_{F,t} - \frac{\tau^w}{\eta + \sigma} \hat{Y}^e_{F,t} \tag{14}$$

where $\Omega_x = \frac{(\eta+\sigma)(1-\tau^w)-\tau^w}{\eta+\sigma}$

Let us note that there is a perfect sectoral integration, $\mathbb{X}_{I,t} = \mathbb{X}_{F,t}$, when taxes are zero or constant. With variable distortionary taxes in the formal sector we have $\Omega_x \neq 1$, and it can be either positive or negative. When $\mathbb{X}_{F,t}$ increases, it then pushes upward $\mathbb{X}_{I,t}$ because the marginal cost increases. But this decreases taxes, which in turn decreases the marginal cost all the more that η and σ are low, so the increase in $\mathbb{X}_{I,t}$ is lower than the increase in $\mathbb{X}_{F,t}$. The higher is the size of the informal sector (N_I) the lower is Ω_x, and therefore the weaker is the positive co-movement across sectoral output gaps. More precisely, the informal sector is less volatile than the formal sector due to the counter-cyclical behavior of the distortive tax in the latter sector.

The Phillips curve

From the optimal pricing program, it is possible to derive the following traditional expression for the dynamics of inflation (See Appendix A2 for derivation):

$$\hat{\pi}_t = \Upsilon \hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} \tag{15}$$

where $\Upsilon = \frac{(1-\omega\beta)(1-\omega)}{\omega}$ and $\hat{\phi}_t$ is the real marginal cost equal to $\hat{P}_t^Y = \hat{\phi}_{F,t} = \hat{\phi}_{I,t}$. The parameter Υ is decreasing in the degree of price rigidity, ω. Thus the higher is the price rigidity, the less sensitive is inflation to changes from the marginal cost.

The expression of the marginal cost can be obtained either from (11) and (14) or from (12) and (13):

$$\hat{\phi}_t = \kappa \mathbb{X}_{F,t} - c p_t \tag{16}$$

where

$\kappa = (\eta + \sigma)(1 - \tau^w) - \tau^w > 0, \quad \frac{\partial \kappa}{\partial N_I} < 0$

$cp_t = \frac{\tau^w \hat{Y}^e_{F,t}}{\eta+\sigma}, \quad \frac{\partial cp_t}{\partial N_I} > 0$

This expression of the marginal cost unveils a crucial feature of our economy with informality. It is in the formal sector only, through the tax variation, that there are cost push shocks, denoted as cp_t. Following a negative productivity shock in the formal sector for instance, there is an increase in the marginal cost due to a higher tax rate; this increase is all the more intense when the economy is more informal $\left(\frac{\partial \tau^w}{\partial N_I} > 0\right)$.

The parameter κ represents the elasticity of the real marginal cost with respect to the formal output gap. As it is traditional in a NK framework this elasticity is small when there are large convexities on preferences (small σ and η). But in the formal sector, as the marginal cost depends on the marginal tax, the coefficient κ also depends on the informality size. It is lowered by the informality size through the feedback effect of tax variation on the marginal cost $\left(\frac{\partial \kappa}{\partial N_I} < 0\right)$.

Combining Equations (15) and (16), the Phillips curve can then be written as follows

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Upsilon \kappa \mathbb{X}_{F,t} - \Upsilon cp_t \tag{17}$$

From the previous discussions, it is straightforward that an economy with larger informality faces higher cost-push shocks on inflation. The Central Bank must decide whether to accept a higher level of inflation volatility or instead stabilizing inflation at the expense of allowing for more fluctuations in the *welfare relevant formal output gap*. In this economy, the sacrifice ratio in terms of formal output gap is particularly high, meaning that stabilizing inflation through formal output gap adjustments is particularly costly in such economies.

IS curve

By log-linearizing the Euler equation (4), we obtain:

$$\hat{Y}_{F,t} = E_t\left[\hat{Y}_{F,t+1}\right] - \frac{1}{\sigma}\hat{i}_t + \frac{1}{\sigma}E_t\left[\hat{\pi}_{t+1}\right]$$

Expressing this function in terms of the aggregate welfare output gap $\left(\hat{Y}_t - \hat{Y}_t^e = \mathbb{X}_t\right)$, we get the aggregate IS curve (See Appendix A3 for derivation):

$$\mathbb{X}_t = E_t\left[\mathbb{X}_{t+1}\right] - \frac{\theta^m}{\sigma}\left[\hat{i}_t - \hat{r}_t^n - E_t\left[\hat{\pi}_{t+1}\right]\right] + \left(\theta^e - \theta^m\right)\frac{1}{\sigma}\hat{r}_t^n \tag{18}$$

where $\hat{r}_t^n$ is the natural interest rate, $\theta^m = \frac{Y_F}{Y}$ and $\theta^e = \frac{Y_F^e}{Y^e}$.

The primary impact of the interest rate on the aggregate demand depends on the formal output share, as households only in the formal sector have access to the financial market. Equation 18 shows that an increase in the size of the informal sector (i.e. a decrease in the formal output share θ^m) decreases the elasticity of aggregate demand to real interest rate, making the monetary policy less effective in containing demand. As the share of the informal sector tends to 1, θ^m decreases towards zero, and also $\theta^e \simeq \theta^m$. Under this scenario, monetary policy is ineffective when nobody has access to the financial market.

4 Optimal monetary policy

4.1 The welfare loss function

The second-order Taylor approximation of the aggregate utility function around the steady state(c_F, c_I, h_F, h_I) yields to the following welfare loss function $\mathbb{W}$ (see Appendix A4 for derivation):

$$\mathbb{W} = E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}\left[\frac{\theta}{\Upsilon}\pi_t^2 + (\eta+\sigma)\left(N_F\mathbb{X}_{F,t}^2 + (1-N_F)\,\mathbb{X}_{I,t}^2\right)\right] - N_F\tau^w\mathbb{X}_{F,t}\right) \tag{19}$$

Welfare losses are expressed in terms of welfare-equivalent permanent loss in consumption, measured as a fraction of steady state consumption. As it is traditional in a NK framework, the weight of inflation volatility is increasing in θ, the elasticity of substitution among goods, and decreasing in the degree of price stickiness ω. An increase in ω will increase the degree of price dispersion resulting from a deviation from a nil inflation. Additionally, the effect of any given price dispersion in the welfare losses will increase with the elasticity of substitution across goods (θ).

The weight associated with the two sectoral *welfare-relevant* output gap volatilities in the loss function is increasing with η and σ, which determine the curvature of the utility function. The higher those parameters, the higher the change in marginal rate of substitution between consumption and hours compared with the change in the marginal cost.

The presence of the linear term $\mathbb{X}_{F,t}$ in the welfare loss function (19) implies that any increase in the formal output gap decreases the welfare losses, because the (flexible-price) equilibrium formal output is below its efficient level. This term measures the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor in the formal sector, and hence the inefficiency of the formal sector steady-state generated by the tax distortion. Note that this term is weighted by the level of taxes and the share of the formal sector. The presence of this term will give rise to the traditional inflation bias when the monetary policy is discretionary.

4.2 Optimal discretionary policy

In this section we characterize the optimal monetary policy under discretion. The Central Bank chooses π_t, $\mathbb{X}_{F,t}$, and $\mathbb{X}_{I,t}$ for a sequence of given expected inflation levels in order to minimize the welfare function subject to the Phillips curve (17) and to the sectoral integration curve (14). The first order conditions of this optimization problem are as follows:

$$[\hat{\pi}_t] \qquad \frac{\theta}{\Upsilon}\pi_t = \lambda_1$$

$$[\mathbb{X}_{F,t}] \qquad (\eta+\sigma)\frac{Y_F}{Y}\mathbb{X}_{F,t} - \frac{Y_F}{Y}\tau^w + \lambda_1\Upsilon\kappa + \lambda_2\Omega_x = 0$$

$$[\mathbb{X}_{I,t}] \qquad (\eta+\sigma)\frac{Y_I}{Y}\mathbb{X}_{I,t} - \lambda_2 = 0$$

whereλ_1 and λ_2 are the Lagrangian multipliers associated to (14) and (17) respectively. Reorganizing these equations, we get:

$$(\eta+\sigma)\left(N_F\mathbb{X}_{F,t} + \Omega_x\left(1-N_F\right)\mathbb{X}_{I,t}\right) - N_F\tau^w + \theta\kappa\pi_t = 0 \qquad (20)$$

First of all, as already noted, there is a positive bias in the inflation level under a discretionary regime. Note that this term is weighted by the level of τ^w and the share of the formal sector. The higher the distortion created by the tax level (when informality is widespread), the further the formal sector from its efficient level. However, the larger the formal sector, the higher the impact on total welfare loss. More informality creates a higher tax level but reduces the scope of this distortion specific to the formal sector. By combining Equations (10), (5) and (6) at the steady state we obtain $\tau^w N_F = \frac{T_F}{A_F h_F}$. From this expression it is straightforward that the inflation bias increases with the size of the informal sector.

Moreover, Equation (20) summarizes the optimal trade-off between inflation and the policy-relevant aggregate output gap defined as $(N_F \mathbb{X}_{F,t} + \Omega_x (1 - N_F) \mathbb{X}_{I,t})$. As in the traditional NK model, the lower θ is and the higher the curvature coefficients (η and σ) are, the higher the relative volatility of inflation should be. This effect is independent of the informality size.

The first insight about how informality matters for the volatility trade-off comes from the analysis conducted above about the Phillips curve: the larger the size of the informal sector is, the lower κ is, and the less the monetary authorities should intervene to stabilize inflation at the expense of the formal output gap volatility. In order to evaluate all the welfare losses due to output gap volatilities, the destabilizing effect on the informal output gap must be accounted for through the integration condition. The parameter Ω_x determines how the informal sector is destabilized by the central bank's intervention. It defines the policy-relevant aggregate output gap $(N_F \mathbb{X}_{F,t} + \Omega_x (1 - N_F) \mathbb{X}_{I,t})$, which ultimately matters in the inflation-activity trade-off, as it is the relevant measure of the welfare loss due to the sectoral output gap variations. This measure is weighted by the relative size of the two sectors, as expected, but also by the sectoral integration parameter Ω_x. The larger the informal sector, the lower Ω_x, the lower the policy-relevant aggregate output volatility for a given output volatility in the formal sector, which turns out to be ultimately less costly to stabilize inflation. When Ω_x is close to 0, only the volatility in the formal sector is source of welfare loss.

Therefore, an increase in the informality size has two opposite effects on inflation volatility: an increase through the sacrifice ratio and a decrease through a lower sectoral integration. The aggregate effect on inflation volatility relative to output volatility would depend on which effect dominates.

5 Quantitative analysis

This section quantifies the effect of a productivity shock, given numerical values for the model's parameters. In the baseline calibration of the model a period t corresponds to a quarter, the discount rate is assumed $\beta = 0.988$, which implies a real interest rate of about 5 percent. We also assume log utility function ($\sigma = 1$), a unitary Frisch elasticity of labor supply ($\eta = 1$), and the elasticity of substitution between varieties $\theta = 6$. These are values commonly found in the business cycle literature (Galí, 2008). In addition, following the empirical evidence found in Galí, Gertler, and López-Salido (2001), Sbordone (2002), and Galí (2008) we set the probability for a firm of not changing prices equal to $\omega = 2/3$, which implies an average price duration of three quarters. N_I is assumed to be equal to 0.5, since, according to the ILO (2018), informal employment accounts for more than half of non-agricultural employment in most developing countries. Transfers to households in the formal sector are calibrated in order to obtain tax revenue of 15 percent of the Gross Domestic Product (GDP) at the steady-state. According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 to 20 percent of GDP, while the average for high-income countries is more like 40 percent. Finally, we assume an auto-regressive coefficient of the productivity shock equal to $\rho = 0.9$.Table 1 summarizes the parameter values of the model.

Table 1. Benchmark model calibration

Description	**Symbol**	**Value**
Discount rate	β	0.988
Inverse of the intertemporal elasticity of substitution	σ	1
Elasticity of labor supply	η	1
Probability for a firm of not changing prices	ω	2/3
Elasticity of substitution between varieties	θ	6
Size of the informal sector	N_I	0.5
Transfers to households in the formal sector	T_F	0.136
Auto-regressive coefficient	ρ	0.9

Figure 1 shows the optimal response o inflation, formal, informal, and total welfare-relevant output gap, to a 1% standard deviation negative productivity shock. We consider different rates of informality, which are indicated as follows: a size of the informal sector equal to 0.5, $N_I = 0.5$ (solid line), $N_I = 0.3$ (dashed line), and $N_I = 0.1$

(dotted line). In all cases, inflation and informal output gap increase after an aggregate negative productivity shock, while formal output gap and total output gap decrease. In particular, note that when the size of the informal sector is large, $N_I = 0.5$, the optimal policy implies a larger increase in inflation, compared with the case when the size of the informal sector is lower $N_I = 0.3$, and $N_I = 0.1$. Additionally, it is worth noticing from Figure 1 that the higher the size of the informal sector, the lower the decrease of the total welfare-based output gap after a negative productivity shock. These results suggest that an increase in the size of the informal increases inflation volatility. The fact that an increase in the informal sector increases the sacrifice ratio, implies that the monetary authorities should intervene less to stabilize inflation at the expense of the formal output gap volatility.

Figure 1. Optimal responses to a negative productivity shock

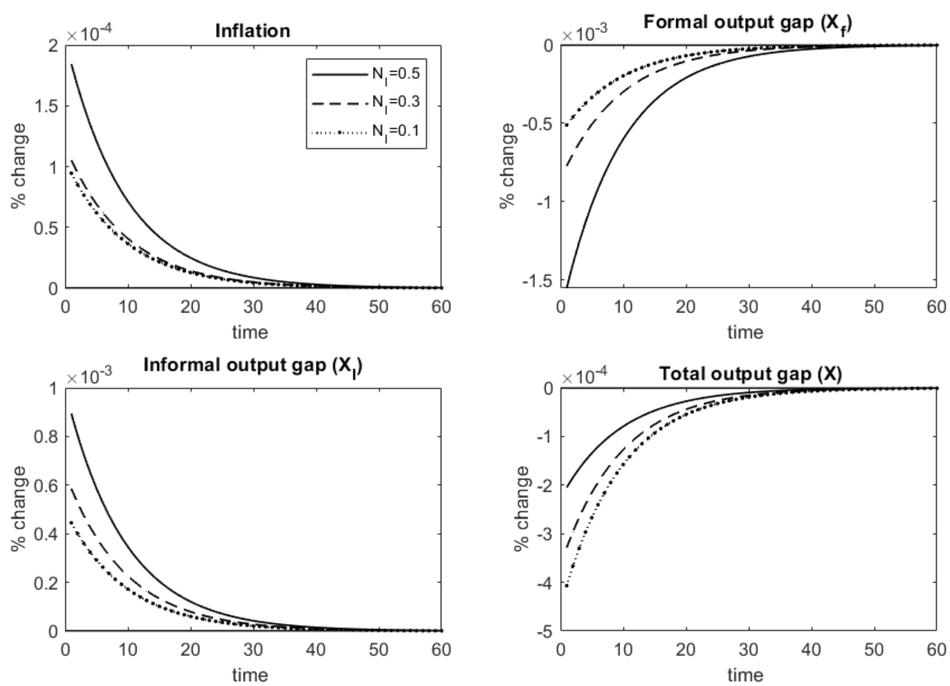
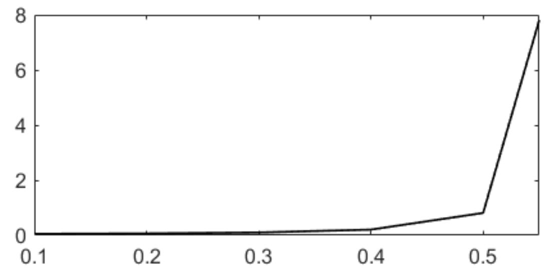


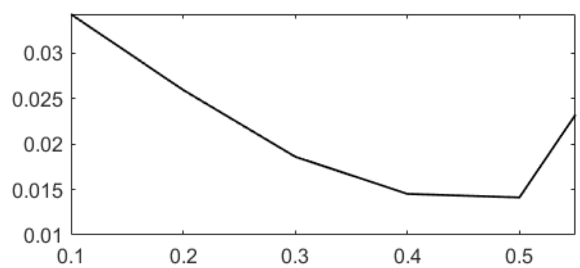
Figure 2 shows the ratio of inflation and formal welfare-relevant output gap volatility for different sizes of the informal sector under the optimal policy. Note that this ratio increases with the size of the informal sector. Similarly, Figure 3 represents the optimal ratio of inflation and formal welfare-relevant output gap volatility for different values of N_I, which initially decreases with the size of the informal sector and then increases when the size of the informal sector is bigger than 0.5. This result suggests that economies with a larger informal sector should stabilize less inflation relative to the total welfare-based output gap.

Figure 2. Ratio of inflation volatility and Total welfare-relevant output gap volatility



Size of the informal sector (N_I)

Figure 3. Ratio of inflation volatility and Formal welfare-relevant output gap volatility



Size of the informal sector (N_I)

6 Conclusion

The focus of this paper is the study of the implications for the Central Bank and the optimal design of monetary policy in economies with a large informal sector. Our results can be summarized as follows. First of all, we find that informality amplifies cost-push shocks on inflation. Secondly, the sacrifice ratio increases with the weight of the informal sector, which would lead to recommending less inflation stability in economies with higher levels of informality. Thirdly, however, in this type of economy, the policy-relevant output gap, which takes into account the degree of the sectoral linkages is less volatile for any level of inflation volatility. Finally, the inflation bias increases with the size of the informal sector.

Therefore, we find that an increase in the informality size has two opposite effects on inflation volatility: an increase through the sacrifice ratio and a decrease through a lower sectoral integration. The aggregate effect on inflation volatility relative to output volatility would depend on which effect dominates. By simulation, we show that economies with a larger informal sector should stabilize less inflation relative to the total output gap.

If these results may be specific to the simple framework considered, we think that our paper is quite general about the key factors at stake in the implications of informality for optimal monetary policy. First of all, our results emphasize the impact of informality on steady-state structural inefficiencies (the gap between the natural and efficient output levels), which depends on both the sectoral location of the inefficiencies and how they are linked to the informality size. Secondly, they unveil the impact of informality on the sacrifice ratio in terms of the policy-relevant output gap, which brings together the Phillips curve sacrifice ratio and the sector integration degree. We think that any more complex frameworks could be analyzed in the same vein by identifying the sector where the structural inefficiencies are, how they are affected by the size of the informal sector, and what the strength of sectoral links in the business cycle is.

Let us examine the case of a model that introduces search frictions only in the formal sector. This adds inefficiencies in the formal sector. If the size of the informal sector increases, there is less employment in the formal sector, which reduces the cost of search frictions at the steady-state. In this model, a more informal economy could display a lower inflationary bias. On the other hand, the formal sector remains the only source of cost-push, and only a more in-depth analysis of the model would make it possible to understand the impact of an increase in informality on the magnitude of cost-push shocks and on the sacrifice ratio in the formal sector. Intuitively, a smaller formal sector decreases the elasticity of the vacancy costs and then increases the sacrifice ratio. In addition, a lower sectoral correlation can be expected when the hiring elasticity to vacancies is greater in the formal sector.

Another important insight brought about by our analysis is that the monetary policy should not target only the formal sector. So only considerations related to informational issues in the measurement of the informal sector could lead to recommend to favor the formal output gap in the monetary policy rule. But the estimated sacrifice ratio of the Phillips curve in the formal sector would be a wrong indicator of the optimal trade-off between inflation and output gap, which would lead to a failure in stabilizing the inflation volatility enough.

7 Appendix

7.1 Appendix A1: Sectoral integration curve

The law of one price implies $\phi_{f,t} = \phi_{I,t}$. This implies a relationship between the sectoral outputs as a sectoral integration condition.

Let us define the marginal cost:

$$\phi_{S,t} = \frac{W_{S,t}}{A_{S,t}P_t} \qquad S = F, I$$

Let us reconsider the expression of the marginal cost in terms of the welfare-relevant output gap.

From (5) and (12)

$$h_{F,t}^{\eta} = (c_{Ft})^{-\sigma} \left(1 - \tau_t^w\right) A_{F,t}\phi_{F,t}$$

$$h_{I,t}^{\eta} = (c_{It})^{-\sigma} A_{I,t}\phi_{I,t}$$

We obtain, after log-linearizing the previous two equations, the following equations:

$$\hat{\phi}_{I,t} = (\eta + \sigma)\left(\hat{\mathbb{Y}}_{I,t}\right) - (\eta + 1)\hat{\mathbb{A}}_{I,t}$$

and

$$\hat{\phi}_{F,t} = (\eta + \sigma)\left(\hat{\mathbb{Y}}_{f,t}\right) + \frac{\tau^w}{1 - \tau^w}\hat{\tau}_t^w - (\eta + 1)\hat{\mathbb{A}}_{f,t}$$

It is also possible to write the same equations for the natural rate equilibrium and the first-best equilibrium:

$$0 = (\eta + \sigma)\left(\hat{\mathbb{Y}}_{I,t}^e\right) - (\eta + 1)\hat{\mathbb{A}}_{I,t}$$

$$0 = (\eta + \sigma)\left(\hat{\mathbb{Y}}_{f,t}^e\right) - (\eta + 1)\hat{\mathbb{A}}_{f,t}$$

$$(\eta + \sigma)\left(\hat{\mathbb{Y}}_{I,t}^e\right) = (\eta + 1)\hat{\mathbb{A}}_{I,t}$$

$$(\eta + \sigma)\left(\hat{\mathbb{Y}}_{f,t}^e\right) = (\eta + 1)\hat{\mathbb{A}}_{f,t}$$

therefore

$$\hat{\phi}_{I,t} = (\eta + \sigma)(\mathbb{X}_{I,t})$$

and

$$\hat{\phi}_{F,t} = (\eta + \sigma)(\mathbb{X}_{F,t}) + \frac{\tau^w}{1-\tau^w}\hat{\tau}_t^w$$

where $\mathbb{X}_{S,t} = \hat{Y}_{S,t} - \hat{Y}_{S,t}^e$ is the *welfare-relevant* output gap in sector S, i.e. the deviation between the actual output $\hat{Y}_{S,t}$ and its efficient level $\hat{Y}_{S,t}^e$.

From Equation (9), $\hat{\phi}_{F,t} = \hat{\phi}_{I,t}$. This sectoral integration condition implies a relationship between the sectoral output gaps:

$$(\eta + \sigma)(\mathbb{X}_{F,t} - \mathbb{X}_{I,t}) = -\frac{\tau^w}{1-\tau^w}\hat{\tau}_t^w$$

Replacing the expression of the tax variation $\hat{\tau}_t^w = -\hat{\phi}_{F,t} - \hat{A}_{F,t} - \hat{h}_{F,t}$, into the previous equation we obtain:

$$\hat{\phi}_{F,t} = (\eta + \sigma)\mathbb{X}_{F,t} - \frac{\tau^w}{1-\tau^w}\left(\hat{\phi}_{F,t} + \hat{A}_{F,t} + \hat{h}_{F,t}\right)$$

reorganizing

$\hat{\phi}_{F,t} = (\eta + \sigma)\mathbb{X}_{F,t} - \frac{\tau^w}{1-\tau^w}\left(\hat{\phi}_{F,t} + \hat{Y}_{F,t}\right)$

$\frac{(1-\tau^w)}{\tau^w}\left(1 + \frac{\tau_t^w}{1-\tau^w}\right)\hat{\phi}_{F,t} = \frac{(1-\tau^w)(\eta+\sigma)}{\tau^w}\mathbb{X}_{F,t} - \left(\hat{Y}_{F,t}\right)$

$\frac{(1-\tau^w)}{\tau^w}\left(1 + \frac{\tau_t^w}{1-\tau^w}\right)\hat{\phi}_{F,t} = \frac{(1-\tau^w)(\eta+\sigma)}{\tau^w}\mathbb{X}_{F,t} - \left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^e + \hat{Y}_{F,t}^e\right)$

$\frac{1}{\tau^w}\hat{\phi}_{F,t} = \frac{(1-\tau^w)(\eta+\sigma)}{\tau^w}\mathbb{X}_{F,t} - \left(\mathbb{X}_{F,t} + \hat{Y}_{F,t}^e\right)$

$\frac{1}{\tau^w}\hat{\phi}_{F,t} = \left(\frac{(1-\tau^w)(\eta+\sigma)}{\tau^w} - 1\right)\mathbb{X}_{F,t} - \hat{Y}_{F,t}^e$

$\hat{\phi}_{F,t} = ((\eta + \sigma)(1 - \tau^w) - \tau^w)\mathbb{X}_{F,t} - \tau^w\hat{Y}_{F,t}^e$

Therefore, the marginal cost in both sector can be written as follows

$$\hat{\phi}_{F,t} = ((\eta + \sigma)(1 - \tau^w) - \tau^w)\mathbb{X}_{F,t} - \tau^w\hat{Y}_{F,t}^e \tag{21}$$

$$\hat{\phi}_{I,t} = (\eta + \sigma)\mathbb{X}_{I,t} \tag{22}$$

Finally, with $\hat{\phi}_{F,t} = \hat{\phi}_{I,t}$, the sectoral integration condition takes the from:

$$\mathbb{X}_{I,t} = \Omega_x \mathbb{X}_{F,t} - \frac{\tau^w}{\eta + \sigma} \hat{Y}^e_{F,t} \tag{23}$$

where $\Omega_x = \frac{(\eta+\sigma)(1-\tau^w)-\tau^w}{\eta+\sigma}$

7.2 Appendix A2: The Phillips curve

From the the first-order condition associated with the optimal pricing program we obtain:

$$\frac{P^*_t}{P_t} = \frac{E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\sigma} \omega^j \left[\phi_{t+j} \left(\frac{P_t}{P_{t+j}} \right)^{-\theta} \right]}{E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\sigma} \omega^j \left[\left(\frac{P_t}{P_{t+j}} \right)^{1-\theta} \right]} \tag{24}$$

log-linearizing (24) around the steady state, and denoting $P^E_t = \frac{P^*_t}{P_t}$ we have that

$$P^E_t E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\sigma} \omega^j \left[\left(\frac{P_t}{P_{t+j}} \right)^{1-\theta} \right] = E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\sigma} \omega^j \left[\phi_{t+j} \left(\frac{P_t}{P_{t+j}} \right)^{-\theta} \right]$$

$$\hat{p}^E_t \left(\frac{c^{1-\sigma}}{1-\omega\beta} \right) + (\theta - 1)\, c^{1-\sigma} E_t \sum_{j=0}^{\infty} (\omega\beta)^j \left(\hat{P}_{t+j} - \hat{P}_t \right) + (1-\sigma) c^{1-\sigma} E_t \sum_{j=0}^{\infty} (\omega\beta)^j \, \hat{c}_{t+j}$$
$$= (1-\sigma)\phi c^{1-\sigma} E_t \sum_{j=0}^{\infty} \beta^j \omega^j \hat{c}_{t+j} + \phi c^{1-\sigma} E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{\phi}_{t+j} \right] + \theta \phi c^{1-\sigma} E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{P}_{t+j} - \hat{P}_t \right]$$

reorganizing previous equation

$$\left(\frac{\hat{p}^E_t}{1 - \omega\beta} \right) = E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{\phi}_{t+j} \right] + E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{P}_{t+j} - \hat{P}_t \right]$$

$$\hat{p}^E_t = (1 - \omega\beta)\, E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{\phi}_{t+j} + \hat{P}_{t+j} - \hat{P}_t \right]$$

then, with $\hat{p}^E_t = \hat{P}^*_t - \hat{P}_t$

$$\hat{P}_t^* = (1-\omega\beta)\, E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{\phi}_{t+j} + \hat{P}_{t+j}\right]$$

Recursively

$$\hat{P}_t^* = \hat{p}_t^E + \hat{P}_t = (1-\omega\beta)\left[\hat{\phi}_t + \hat{P}_t\right](1-\omega\beta)\,\omega\beta E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[\hat{\phi}_{t+j+1} + \hat{P}_{t+j+1}\right]$$

$$\hat{p}_t^E = (1-\omega\beta)\,\hat{\phi}_t + \omega\beta\left(E_t\hat{P}_{t+1}^E + E_t\hat{\pi}_{t+1}\right) \tag{25}$$

Log-linearizing the the price Index we obtain:

$$0 = \omega\left(\hat{P}_{t-1} - \hat{P}_t\right) + (1-\omega)\left(\hat{P}_t^E\right)$$

$$\hat{P}_t^E = \frac{\omega}{(1-\omega)}\left(\hat{\pi}_t\right) \tag{26}$$

replacing (26) into (25)

$$\frac{\omega}{(1-\omega)}\hat{\pi}_t = (1-\omega\beta)\,\hat{\phi}_t + \omega\beta\left(E_t\frac{\omega}{(1-\omega)}\hat{\pi}_{t+1} + E_t\hat{\pi}_{t+1}\right)$$

$$\hat{\pi}_t = \frac{(1-\omega\beta)\,(1-\omega)}{\omega}\hat{\phi}_t + \beta E_t\hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \Upsilon\hat{\phi}_t + \beta E_t\hat{\pi}_{t+1}$$

where $\Upsilon = \frac{(1-\omega\beta)(1-\omega)}{\omega}$ and $\hat{\phi}_t$ is the real marginal cost equal to $\hat{P}_t^Y = \hat{\phi}_{F,t} = \hat{\phi}_{I,t} = \hat{\phi}_t$.

The expression of the marginal cost can be obtained either from (22) and (14) or from (21).

$$\hat{\phi}_t = \kappa \mathbb{X}_{F,t} + cp_t$$

where

$\Omega_x = \frac{(\eta+\sigma)(1-\tau^w)-\tau^w}{\eta+\sigma}$

$\kappa = (\eta + \sigma)(1 - \tau^w) - \tau^w > 0, \quad \frac{\partial \kappa_f}{\partial N_F} > 0$

$cp_t = -\tau^w \hat{Y}^e_{F,t}, \quad \frac{\partial cp_t}{\partial N_F} > 0$

The Phillips curve can then be written as follows:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Upsilon \kappa \mathbb{X}_{F,t} + \Upsilon cp_t \tag{27}$$

7.3 Appendix A3: IS curve

Now log-linearizing the Euler equation (4) we obtain:

$$\hat{Y}_{F,t} = E_t\left[\hat{Y}_{F,t+1}\right] - \frac{1}{\sigma}\hat{i}_t + \frac{1}{\sigma}E_t\left[\hat{\pi}_{t+1}\right] \tag{28}$$

on the other side, the social planner Euler equation can be represented as follows

$$\hat{Y^e}_{F,t} = E_t\left[\hat{Y^e}_{F,t+1}\right] - \frac{1}{\sigma}\hat{r}^n$$

where $\hat{r}^n$ is the natural interest rate.

From the last two equations we can find ans expression for $\hat{Y}_{F,t} - \hat{Y}^e_{F,t}$, this is:

$$\hat{Y}_{F,t} - \hat{Y}^e_{F,t} = E_t\left[\hat{Y}_{F,t+1} - \hat{Y}^e_{F,t+1}\right] - \frac{1}{\sigma}\left[\hat{i}_t - \hat{r}^n_t - E_t\left[\hat{\pi}_{t+1}\right]\right]$$

$$\mathbb{X}_{F,t} = E_t\left[\mathbb{X}_{F,t+1}\right] - \frac{1}{\sigma}\left[\hat{i}_t - \hat{r}^n_t - E_t\left[\hat{\pi}_{t+1}\right]\right]$$

Multiplying (28) by $\frac{Y_F}{Y}$

$$\frac{Y_F}{Y}\hat{Y}_{F,t} = E_t\left[\frac{Y_F}{Y}\hat{Y}_{F,t+1}\right] - \frac{1}{\sigma}\frac{Y_F}{Y}\hat{i}_t + \frac{1}{\sigma}\frac{Y_F}{Y}E_t\left[\hat{\pi}_{t+1}\right]$$

adding $\frac{Y_I}{Y}\hat{Y}_{I,t}$ to both sides

$$\frac{Y_F}{Y}\hat{Y}_{F,t} + \frac{Y_I}{Y}\hat{Y}_{I,t} = E_t\left[\frac{Y_F}{Y}\hat{Y}_{F,t+1}\right] + \frac{Y_I}{Y}\hat{Y}_{I,t} - \frac{1}{\sigma}\frac{Y_F}{Y}\hat{i}_t + \frac{1}{\sigma}\frac{Y_F}{Y}E_t\left[\hat{\pi}_{t+1}\right]$$

given that we have that for the informal sector $E_t\left[\hat{Y}_{I,t+1}\right] = \hat{Y}_{I,t+1}$, previous equations becomes

$$\hat{Y}_t = E_t\left[\frac{Y_F}{Y}\hat{Y}_{F,t+1} + \frac{Y_I}{Y}\hat{Y}_{I,t+1}\right] - \frac{1}{\sigma}\frac{Y_F}{Y}\hat{i}_t + \frac{1}{\sigma}\frac{Y_F}{Y}E_t\left[\hat{\pi}_{t+1}\right]$$

$$\hat{Y}_t = E_t\left[\hat{Y}_{t+1}\right] - \frac{Y_F}{Y}\frac{1}{\sigma}\left(\hat{i}_t - E_t\left[\hat{\pi}_{t+1}\right]\right)$$

Expressing this function in terms of the aggregate welfare output gap ($\hat{Y}_t - \hat{Y}_t^e = \mathbb{X}_t$), we get the aggregate IS curve:

$$\hat{Y}_t - \hat{Y}_t^e = \left(E_t\left[\hat{Y}_{t+1}\right] - \frac{Y_F}{Y}\frac{1}{\sigma}\left(\hat{i}_t - E_t\left[\hat{\pi}_{t+1}\right]\right)\right) - \left(E_t\left[\hat{Y}_{t+1}^e\right] - \frac{Y_F^e}{Y^e}\frac{1}{\sigma}\hat{r}_t^n\right)$$

$$\hat{Y}_t - \hat{Y}_t^e = E_t\left[\hat{Y}_{t+1} - \hat{Y}_{t+1}^e\right] - \frac{Y_F}{Y}\frac{1}{\sigma}\left(\hat{i}_t - \hat{r}_t^n - E_t\left[\hat{\pi}_{t+1}\right]\right) + \left(\frac{Y_F^e}{Y^e} - \frac{Y_F}{Y}\right)\frac{1}{\sigma}\hat{r}_t^n$$

we get the aggregate IS curve:

$$\mathbb{X}_t = E_t\left[\mathbb{X}_{t+1}\right] - \frac{1}{\sigma}\frac{Y_F}{Y}\left(\hat{i}_t - \hat{r}_t^n - E_t\left[\hat{\pi}_{t+1}\right]\right) + \left(\frac{Y_F^e}{Y^e} - \frac{Y_F}{Y}\right)\frac{1}{\sigma}\hat{r}_t^n$$

$$\mathbb{X}_t = E_t\left[\mathbb{X}_{t+1}\right] - \frac{\theta^m}{\sigma}\left[\hat{i}_t - \hat{r}_t^n - E_t\left[\hat{\pi}_{t+1}\right]\right] + \left(\theta^e - \theta^m\right)\frac{1}{\sigma}\hat{r}_t^n$$

where $\theta^m = \frac{Y_F}{Y}$ and $\theta^e = \frac{Y_F^e}{Y^e}$

7.4 Appendix A4: The welfare loss function

The second order approximation of the utility function in the formal and in the informal sector around an steady state yields, for each sector

$$U_{F,t}-U_F \simeq U_cC_I\left(\frac{c_{F,t}-c_F}{c_F}\right)+U_{h_F}h_F\left(\frac{h_{F,t}-h_F}{h_F}\right)+\frac{1}{2}U_{c_Fc_F}C_F^2\left(\frac{c_{Ft}-c_F}{c_F}\right)^2+\frac{1}{2}U_{h_Fh_F}h_F^2\left(\frac{h_{F,t}-h_F}{h_F}\right)^2$$

$$U_{I,t}-U_I \simeq U_cC_I\left(\frac{c_{I,t}-c_I}{c_I}\right)+U_{h_I}h_I\left(\frac{h_{I,t}-h_I}{h_I}\right)+\frac{1}{2}U_{c_Ic_I}C_I^2\left(\frac{c_{I,t}-c_I}{c_I}\right)^2+\frac{1}{2}U_{h_Ih_I}h_I^2\left(\frac{h_{I,t}-h_I}{h_I}\right)^2$$

We use the following second-order approximation of relative deviations in terms of log deviations

:$\frac{z_t-z}{z} \simeq \hat{z}_t+\frac{1}{2}\hat{z}_t^2$, where $\hat{z}_t = z_t - z$

$$U_{F,t}-U_F \simeq U_cC_F\left(\hat{c}_{F,t}+\frac{1}{2}\hat{c}_{F,t}^2\right)+U_{h_F}h_F\left(\hat{h}_{F,t}+\frac{1}{2}\hat{h}_{F,t}^2\right)+\frac{1}{2}U_{c_Fc_F}C_F^2\left(\hat{c}_{F,t}+\frac{1}{2}\hat{c}_{F,t}^2\right)^2+\frac{1}{2}U_{h_Fh_F}h_F^2\left(\hat{h}_{F,t}\right)^2$$

$$U_{I,t}-U_I \simeq U_cC_I\left(\hat{c}_{I,t}+\frac{1}{2}\hat{c}_{I,t}^2\right)+U_{h_I}h_I\left(\hat{h}_{I,t}+\frac{1}{2}\hat{h}_{I,t}^2\right)+\frac{1}{2}U_{c_Ic_I}C_I^2\left(\hat{c}_{I,t}+\frac{1}{2}\hat{c}_{I,t}^2\right)^2+\frac{1}{2}U_{h_Ih_I}h_I^2\left(\hat{h}_{I,t}\right)^2$$

then

$$U_{F,t}-U_F \simeq U_cC_F\left[\hat{c}_{F,t}+\frac{1-\sigma}{2}\hat{c}_{F,t}^2\right]+U_{h_F}h_F\left(\hat{h}_{F,t}+\frac{1+\eta}{2}\hat{h}_{F,t}^2\right)$$

$$U_{I,t}-U_I \simeq U_cC_I\left[\hat{c}_{I,t}+\frac{1-\sigma}{2}\hat{c}_{I,t}^2\right]+U_{h_I}h_I\left(\hat{h}_{I,t}+\frac{1+\eta}{2}\hat{h}_{I,t}^2\right)$$

with $\hat{y_S} = \hat{c_S}$ and $\hat{h}_{s,t} = \hat{y}_{s,t} - \hat{A}_{s,t} + d_t$ where $d_t = log\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\theta} d_i$ .Galí (2008, p87) shows that d_t is proportional to the cross-sectional variance of relative prices. Therefore, $d_t \simeq \frac{\theta}{2}var_i\left\{p_t(i)\right\}$

Now the period t utility can be writing as follows:

$$U_{F,t} - U_F \simeq U_c C_F \left[\hat{y}_{F,t} + \frac{1-\sigma}{2} \hat{y}_{F,t}^2 \right] + U_{h_F} h_F \left(\hat{y}_{F,t} + \frac{\theta}{2} var_i \{p_t(i)\} + \frac{1+\eta}{2} \left(\hat{y}_{F,t} - \hat{A}_{F,t} \right)^2 \right) + t.i.p$$

$$U_{I,t} - U_I \simeq U_c C_I \left[\hat{y}_{I,t} + \frac{1-\sigma}{2} \hat{y}_{I,t}^2 \right] + U_{h_I} h_I \left(\hat{y}_{I,t} + \frac{\theta}{2} var_i \{p_t(i)\} + \frac{1+\eta}{2} \left(\hat{y}_{I,t} - \hat{A}_{I,t} \right)^2 \right) + t.i.p$$

where $t.i.p$ represents all terms independent of policy.
Equilibrium of the steady stated implies $-\frac{U_{h_F}}{Uc} = MPH_F = (1-\tau^w)\frac{Y_F}{h_F}$, $-\frac{U_{h_I}}{Uc} = MPH_I = \frac{Y_I}{h_I}$, and $Y_s = C_s$.

$$\frac{U_{F,t} - U_F}{U_c C_F} \simeq \left[\hat{y}_{F,t} + \frac{1-\sigma}{2} \hat{y}_{F,t}^2 \right] - (1-\tau^w) \left(\hat{y}_{F,t} + \frac{\theta}{2} var_i \{p_t(i)\} + \frac{1+\eta}{2} \left(\hat{y}_{F,t} - \hat{A}_{F,t} \right)^2 \right) + t.i.p$$

$$\frac{U_{I,t} - U_I}{U_c C_I} \simeq \left[\hat{y}_{I,t} + \frac{1-\sigma}{2} \hat{y}_{I,t}^2 \right] - \left(\hat{y}_{I,t} + \frac{\theta}{2} var_i \{p_t(i)\} + \frac{1+\eta}{2} \left(\hat{y}_{I,t} - \hat{A}_{I,t} \right)^2 \right) + t.i.p$$

under the small distortion assumption (so that the product of τ^w with a second order term can be ignored as negligible),

$$\frac{U_{F,t} - U_F}{U_c C_F} \simeq \left[\hat{y}_{F,t} + \frac{1-\sigma}{2} \hat{y}_{F,t}^2 \right] - \left((1-\tau^w) \hat{y}_{F,t} + \frac{\theta}{2} var_i \{p_t(i)\} + \frac{1+\eta}{2} \left(\hat{y}_{F,t} - \hat{A}_{F,t} \right)^2 \right) + t.i.p$$

$$\frac{U_{I,t} - U_I}{U_c C_I} \simeq \left[\hat{y}_{I,t} + \frac{1-\sigma}{2} \hat{y}_{I,t}^2 \right] - \left(\hat{y}_{I,t} + \frac{\theta}{2} var_i \{p_t(i)\} + \frac{1+\eta}{2} \left(\hat{y}_{I,t} - \hat{A}_{I,t} \right)^2 \right) + t.i.p$$

then

$$\frac{U_{F,t} - U_F}{U_c C_F} \simeq -\frac{1}{2} \left(-2\tau^w \hat{y}_{F,t} + \theta var_i \{p_t(i)\} + (\eta + \sigma) \hat{y}_{F,t}^2 - 2(1+\eta) \left(\hat{y}_{F,t} \hat{A}_{F,t} \right) \right) + t.i.p$$

$$\frac{U_{I,t} - U_I}{U_c C_I} \simeq -\frac{1}{2} \left(\theta var_i \{p_t(i)\} + (\eta + \sigma) \hat{y}_{I,t}^2 - 2(1+\eta) \left(\hat{y}_{I,t} \hat{A}_{I,t} \right) \right) + t.i.p$$

with

$$\hat{Y^e}_{I,t} = \frac{1+\eta}{\eta+\sigma}\hat{A}_{I,t}$$

$$\hat{Y^e}_{F,t} = \frac{1+\eta}{\eta+\sigma}\hat{A}_{F,t}$$

we obtain

$$\frac{U_{F,t}-U_F}{U_c C_F} \simeq -\frac{1}{2}\left(-2\tau^w \mathbb{X}_{F,t} + \theta var_i\left\{p_t(i)\right\} + (\eta+\sigma)\left(\mathbb{X}_{F,t}\right)^2\right) + t.i.p$$

$$\frac{U_{I,t}-U_I}{U_c C_I} \simeq -\frac{1}{2}\left(\theta var_i\left\{p_t(i)\right\} + (\eta+\sigma)\left(\mathbb{X}_{I,t}\right)^2\right) + t.i.p$$

Accordingly, a second-order approximation can be written to the consumer's welfare losses (up to additive terms independent of policy), and expressed as a fraction of steady state consumption as

$$\mathbb{W} = -E_0\sum_{t=0}^{\infty}\beta^t\left[N_F\frac{U_{F,t}-U_F}{U_c C_F} + (1-N_F)\frac{U_{I,t}-U_I}{U_c C_I}\right]$$

$$\mathbb{W} = E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\left[\theta var_i\left\{p_t(i)\right\} + (\eta+\sigma)\left(N_F\left(\mathbb{X}_{F,t}\right)^2 + (1-N_F)\left(\mathbb{X}_{I,t}\right)^2\right) - N_F\tau^w\mathbb{X}_{F,t}\right]\right\}$$

In Woodford (2003, chapter 6) is proved that $\sum_{t=0}^{\infty}\beta^t var_i\left\{p_t(i)\right\} = \frac{\omega}{(1-\beta\omega)(1-\omega)}\sum_{t=0}^{\infty}\beta^t\pi_t^2$

with $\Upsilon = \frac{(1-\beta\omega)(1-\omega)}{\omega}$ we have $\sum_{t=0}^{\infty}\beta^t var_i\left\{p_t(i)\right\} = \frac{1}{\Upsilon}\sum_{t=0}^{\infty}\beta^t\pi_t^2$

Therefore, the second-order Taylor approximation of the aggregate utility function around the steady state(c_F, $c_{I,}$ h_F, h_I) yields to the following welfare loss function $\mathbb{W}$:

$$\mathbb{W} = E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}\left[\frac{\theta}{\Upsilon}\pi_t^2 + (\eta+\sigma)\left(N_F\mathbb{X}_{F,t}^2 + (1-N_F)\mathbb{X}_{I,t}^2\right)\right] - N_F\tau^w\mathbb{X}_{F,t}\right) \tag{29}$$

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