Job Polarization and the Informal Labor Market

Monica A. Gomez Ospina

August 2021

Abstract

This paper analyses the incidence of job polarization in developing and emerging countries, where a substantial fraction of the urban labor force works in the informal sector. I build a general equilibrium model with informality and endogenous occupational choice. Workers in the informal sector do not pay taxes, are less productive, and have the same ability to perform manual tasks. The analytical solution of the model shows that job polarization, driven by a Routine-Biased Technological Change (RBTC), increases the size of the informal sector. Additionally, I find that the share of informal employment in the service sector and the wage inequality at the bottom of the skill distribution decrease with technological progress.

1 Introduction

The polarization of employment in industrialized countries has been a widely studied phenomenon over the last couple of decades. It documents a simultaneous growth in employment and wages of high-skill (problem-solving, creativity, situational adaptability, and in-person interactions) occupations and low-skill (personal services) occupations, compared to middle-skill (production, clerical, and sales) occupations. The main explanation about the drivers of job polarization is the Routine- Biased Technological Change (RBTC) hypothesis, first introduced by Autor et al. (2003), which suggests that technological progress tends to substitute for workers who operate routine tasks. Simultaneously, it increases the relative demand for workers who perform complementary non-routine tasks (abstract and manual tasks). According to Acemoglu and Autor (2011) routine tasks are characteristic of many middle-skilled cognitive and manual jobs (such as bookkeeping, clerical work, repetitive production, and monitoring jobs).

Considering that the core job tasks of these occupations follow precise and well-understood procedures, they can be codified in computer software and performed by machines (or they can be sent electronically outsourced to foreign worksites). As a result, this process of automation and offshoring of routine tasks raises relative demand for workers who can perform a complementary non-routine task, i.e. problem-solving, situational adaptability, creativity, and in-person interactions.

Additional to the *routinization* hypothesis, some authors have also pointed to offshoring as a driver of job polarization. Blinder (2009) states that since many tasks can be automated, they can also be suitable for offshoring to a low-cost producer in a different country, without a deterioration in quality. As a result, international trade in routine tasks may also have a polarizing effect on the labor market of countries that engage in offshoring (Reijnders and de Vries, 2018). Some studies (Goos et al. 2014, Michaels et al. 2014, and Reijnders and de Vries, 2018) use cross-country empirical data to quantify the importance of RBTC and offshoring in explaining job polarization, find that RBTC drives job polarization in almost all countries, with offshoring having a much less significant impact. However, recent detailed country studies, Author et. al, (2015) for the US and Keller and Utar (2016) for Denmark, find that local labor markets with greater exposure to trade competition experience large declines in manufacturing employment.

A sizable body of literature dealing with polarization of the labor market (see Autor et al. 2003, Autor et al. 2006, Autor and Dorn 2013, Goos et al. 2014, Michaels et al. 2014, Feng and Graetz 2015) focus on analyzing this phenomenon in developed countries, specifically in the United States and several European countries. However, the phenomenon of job polarization is not limited to developed countries. According to the World Development Report (2016), there are signs that employment is also polarizing in several low and middle-income countries. This study finds that the average decline in the share of routine employment has been 7.8 percentage points for the period 1995-2012. Reijnders and de Vries (2018) also find evidence of an increase in the share of non-routine jobs in total employment for a group of advanced and major emerging countries during the period 1999-2007. They find that for all these countries, technological change was the main force behind employment changes.

One of the main features of labor markets in emerging economies is the existence of a large informal sector[1]. Informality refers to activities that are outside the regulatory frameworks, most workers in this sector are self-employed, and their income comes from operating small unincorporated enterprises. These include activities such as trading on the streets or in markets; sales of cooked food from kiosks; the transport of people or goods by pedal-power or motorbikes;

---

[1]Informal employment accounts for more than half of non-agricultural employment in most developing countries:around 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States, 50 percent in Latin America, and 30 percent in Europe and Central Asia. In the case of developed countries, only 17 percent of the urban labor force is employed in informal activities (International Labor Office, 2018).

repairing clothes, shoes, or motor scooters; dwelling construction or adding extensions to them; scavenge for reusable waste; or providing a range of personal services like hairdressing, shoe cleaning, street theater, house cleaning, and the like (Blades et al. 2011). In sum, the informal sector can be described as a labor-intensive sector with poor working conditions and relatively lower productivity compared with the formal economy.

The informal sector contributes significantly to employment creation, production, and income generation in developing countries. Nevertheless, a large informal sector has negative consequences for competitiveness and growth and may also be the source of further economic retardation (Loayza 1996, Loayza, Oviedo, and Serven 2004)[2]. But on the other side, empirical evidence also shows that as countries become richer, the share of informal employment falls (see La Porta and Shleifer 2014, Docquier et. al 2017). Elgin and Birinci (2016) use a panel dataset of 161 advanced and emerging market economies over the period from 1950 to 2010 and find an inverted-U relationship between informal sector size and growth of GDP per capita. This result implies that small and large sizes of the informal economy are associated with little growth and medium levels of the size of the informal economy are associated with higher levels of economic growth. As a result, the relationship between informality and growth is not linear and it depends on the stage of development and characteristics of each country. Similarly, Wu and Schneider (2019) use a dataset of 158 countries over the period from 1996 to 2015. They find a robust U-shaped relationship between the shadow economy size and GDP per capita.

Since the job polarization process involves a significant reallocation and mobility of workers in the labor market, it is very likely to produce different performances in the presence of a large informal sector. According to the ILO (2018b) "technology is likely to have both positive and negative effects on informality. Productivity and economic structure influence the income distribution and labor outcomes, including informality. Technology can increase informality via the probable increase in the productivity gaps among economic units, especially when access to technologies is not equal, or via the spreading of new forms of work, especially those where informality is higher. But the question is, can it also reduce informality? " There is not a consensus and enough evidence about the implications of job polarization on informality. More studies, empirical and theoretical, would be required to analyze the effect of job polarization on informal employment and wages.

This paper contributes to this literature by analyzing the incidence of job polarization in developing countries on informality, using a structural model. I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorn (2013). I consider a labor market in which some workers are low-skill while others are high-skill. I assume that there are three sectors in this economy: the goods sector uses capital and employs workers

---

[2]On average, economies with larger informal sectors tend to have lower productivity, slower physical and human capital accumulation, higher poverty and inequality, and smaller fiscal resources (World Bank, 2019)

to perform abstract and routine tasks; the formal service sector employs workers to perform manual tasks, and; the informal service sector employs workers also to perform manual tasks. Workers in the informal service sector can avoid taxation, but are less productive.

A key feature of the model is that households can produce services in the informal sector[3], which are substitutes for services produced in the formal sector, and complement for goods. Additionally, each worker is characterized by a set of skills in performing abstract, routine, and manual tasks. High-skill workers only perform abstract tasks, and low-skill workers can perform both routine and manual tasks. I assume that low-skill workers have the same ability to accomplish manual tasks in the informal sector, while they are heterogeneous in their ability to perform a routine task or a manual task in the formal sector. This feature implies that workers moving from the goods sector to the formal service sector can keep some of their abilities, while the ones moving to the informal sector will have the same ability as all informal workers.

Additionally, the lack of taxation in the informal sector leads to an inefficient reallocation of employment between the formal and informal service sectors. At the same time, it allows the fiscal policy to have an asymmetric effect on the reallocation of labor between these two sectors.

The analytical solution (asymptotic solution) of the model shows that when the elasticity of substitution between capital and routine labor is higher than the elasticity of substitution between goods and services, the constant decrease in prices of automating routine tasks eventually causes low-skill labor flows from routine tasks to manual tasks. In this case, Routine-biased technological change (RBTC), affecting mainly the production of goods, can increase aggregate demand for services and eventually increase employment and wages in service occupations. Additionally, when goods and services are complements, wages also polarize. These conditions for job and wage polarization are the same found by Autor and Dorn (2013), in the case without informality.

I find that employment and wages in both the formal and the informal service sectors increase due to the increased demand for services. The allocation of labor in the service sector depends on the level of labor income taxes, the degree of substitution between the two types of services, and the level of efficiency in the formal service sector. I find that the relative units of efficient labor in the informal sector (as well as the share of informal employment in the service sector) decrease with technological progress. This result is driven by the assumption that workers, previously working in routine tasks, still can use some of their skills when they move to the formal service sector, while all workers in the informal sector have the same ability to accomplish manual tasks. Therefore, the increasing demand for labor in the informal service sector requires a higher increase in wages to compensate for the loss of skills from working in this sector. As a consequence, relative wages in the informal sector increase, as well as their relative prices. It

---

[3]Since informal sector is labor-intensive, I assume that informal workers are employed in manual task occupations

follows that the increase in relative prices of informal services decreases their relative demand and therefore the relative demand for workers in the service sector.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the analytical solution of the model for the labor allocation. Section 4 derives the analytical solution of the model for the relative wages. Section 5 presents the results of the model simulations, and Section 6 concludes.

2 Model

In this section, I develop a general equilibrium model with three sectors and endogenous occupational choice based on Autor and Dorn (2013). The goods sector employs high-skilled workers in abstract jobs L_a, low-skill workers in routine tasks L_r, and Capital K. The service sector is composed of two sectors: formal and informal, which use only unskilled labor in manual tasks, L_{sf} and L_{si} respectively. I assume that high-skilled workers have homogeneous skills at performing abstract tasks, while their skills are heterogeneous in performing routine tasks and manual tasks in the formal sector. As in Autor and Dorn (2013), Routine-biased technological change is modeled as an exogenous fall in the price of capital P_k.

2.1 The goods sector

The goods sector is perfectly competitive and uses abstract tasks L_a, routine tasks L_r, and capital K to produce Y_g units of goods. As in Autor and Dorn (2013), I assume the good sector uses the following technology:

$$Y_g = A_g \left(L_a\right)^{(1-\beta)} X^\beta = A_g \left(L_a\right)^{(1-\beta)} \left[\left(\left(1 - \mu_k\right) L_r\right)^\nu + \left(\mu_k K\right)^\nu\right]^{\frac{\beta}{\nu}}, \tag{1}$$

with μ_k the capital input share, and $\beta \in (0,1)$ and $\nu \in (0,1)$. From equation (1), the elasticity of substitution between abstract labor and the total routine task is equal to 1, while the elasticity of substitution between routine labor and computer capital is $\frac{1}{1-\nu} > 1$. As a result, K is a relative complement to abstract labor and a relative substitute for routine labor.

Firms in the goods sector solve the following maximization problem:

$$Max \; \Pi_t^G = P_g A_g L_a^{1-\beta} X^\beta - P_k K - w_r L_r - w_a L_a. \tag{2}$$

The first-order conditions for problem (2) with respect to abstract labor L_a, routine labor L_r, and capital K respectively are given by

$$P_g (1 - \beta) \frac{Y_g}{L_a} = w_a, \tag{3}$$

$$\kappa_R \frac{Y_g}{L^r} \left(\frac{L_r}{X} \right)^{\nu} = w_r, \tag{4}$$

$$\kappa_k \frac{Y_g}{K} \left(\frac{K}{X} \right)^{\nu} = P_k, \tag{5}$$

where $\kappa_R = \beta \mu_r^{\nu}$ and $\kappa_k = \beta \mu_k^{\nu}$. I have normalized $P_g = 1$.

2.2 The service sector

The service sector uses manual tasks to produce services in a competitive environment. There are two types of firms in this sector: formal and informal. Workers and firms in the formal service sector have to pay labor income taxes, but due to the better employment conditions workers are more productive. Workers in the informal sector can avoid taxes but are less productive.

2.2.1 Formal service sector

The formal service sector uses only manual labor as input. The production function writes:

$$Y_{sf} = A_{sf} L_{sf},$$

where L_{sf} is the total efficient units of manual labor employed in the formal service sector, and A_{sf} is the common labor productivity in this sector.

Firms solve the following maximization problem:

$$Max \ \Pi_{sf} = P_{sf} Y_{sf} - w_{sf} L_{sf},$$

In equilibrium, wage per efficiency unit of labor in the formal service sector is equal to their marginal productivity:

$$w_{sf} = P_{sf} A_{sf}. \tag{6}$$

Since workers in the formal service sector differs in their skill to perform manual task activities, the wage of a worker with η_i^{θ} efficiency units of manual labor is $\eta_i^{\theta} w_{sf}$.

2.2.2 Informal service sector

The informal sector is labor-intensive, and uses only manual labor as input. The total amount of informal services produced in the economy is:

$$Y_{si} = A_{si} L_{si},$$

where L_{si} is the total units of low-skill labor employed in the informal service sector, and A_{si} is the labor productivity in this sector. With $A_{si} < A_{sf}$. I assume that each worker is equally talented in providing low-skilled informal services.

The profit maximization problem writes:

$$Max \; \Pi_{si} = P_{si} Y_{si} - w_{si} L_{si}.$$

The first-order condition with respect to L_{si} implies:

$$w_{si} = P_{si} A_{si}. \tag{7}$$

Note that since everyone has the same skill in this sector, everyone working in the low-skilled informal service sector earns the same wage.

2.3 Capital

Capital is produced and supplied in a competitive framework. As in Autor and Dorn (2013), The production technology of capital is described by

$$K = Y_k \frac{e^{\delta_k t}}{\Theta}, \tag{8}$$

where Y_k is the amount of final goods used to produce capital, $\delta_k > 0$, and $\Theta = e^{\delta_k}$ is an efficiency term. Capital fully depreciates at each period. δ_k represents the growth rate of capital productivity, and t represents the period of time.

The price of capital is equal to its marginal cost.

$$
\begin{aligned}
P_k &= \frac{Y_k}{K} \\
&= e^{-\delta_k(t-1)}.
\end{aligned} \tag{9}
$$

Note that as time passes, the price of capital falls to zero asymptotically.

2.4 Occupational choice

I assume that every member of the household works full-time in one of the three market sectors. Low-skill workers are heterogeneous in their endowment of efficiency units of labor η, which is drawn from a time-invariant distribution $f(\eta)$. The endowment η determines the productivity of each individual in each sector. I assume that η denotes the worker's efficiency units of labor in routine tasks, while η^θ denotes the worker's efficiency units of labor in manual tasks in the formal service sector. Workers in the informal service sector have homogeneous skills in performing manual tasks, each worker in this sector supply a unit mass of manual labor. This assumption implies that a worker with endowment η has individual productivity, measured in efficiency units, of η performing routine tasks, η^θ performing manual tasks in the formal service sector, with $\theta \in (0,1)$, and 1 performing manual tasks in the informal service sector. Note that workers with an endowment $\eta > 1$ are more efficient in the formal service sector than in the informal service sector. This assumption is motivated by the fact that most of the workers in the formal sector work in larger companies with better working conditions, and in most cases, they have access to training programs. While workers in the informal sector are self-employed. Therefore, it is realistic to assume that some workers are more skilled performing manual task activities in the formal sector than in the informal sector.

I assume that workers in the formal service sector (goods and formal services) have to pay labor income taxes, while workers in the informal sector can avoid taxation.

Since any low-skill worker can work in any of the three sectors, it is optimal for each worker to choose the type of work and the sector that provides her with the highest wage. Therefore, it is optimal for an individual i endowed with η_i efficiency units of labor to work in the goods sector only if:

$$\eta_i(1-\tau)w_r \geq max\left[\eta_i^\theta(1-\tau)w_{sf}, w_{si}\right].$$

$$\eta^* = \left(\frac{w_{sf}}{w_r}\right)^{\frac{1}{1-\theta}}. \tag{10}$$

Workers whose efficiency level is lower than η^* have to decide whether to work in the formal service sector or the informal service sector. Then, it is optimal for an individual to work in the formal service sector only if:
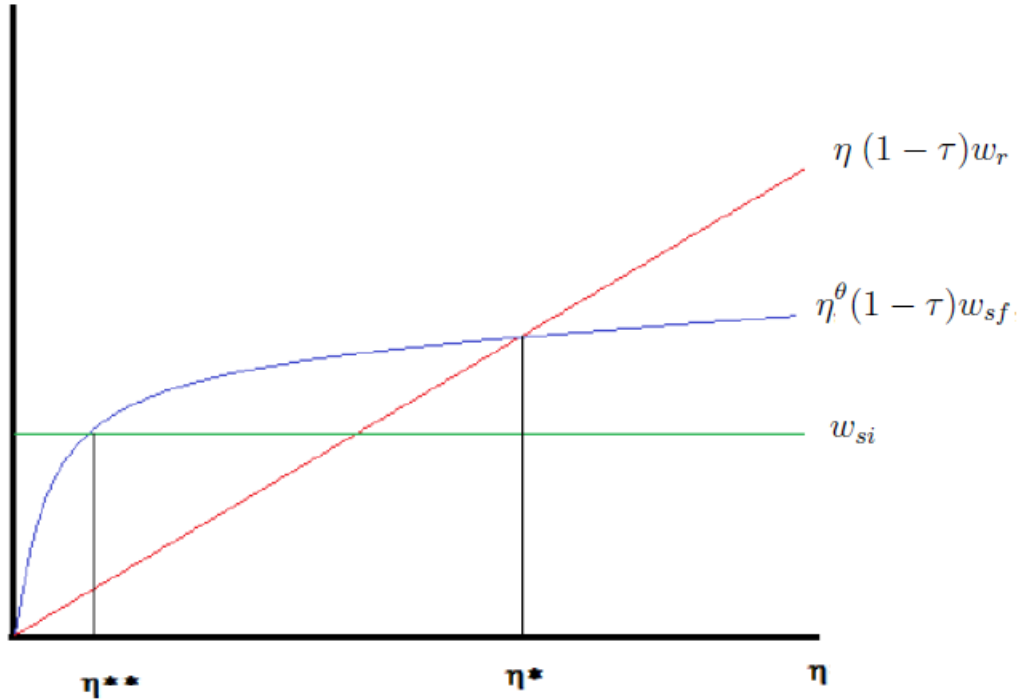
$$\eta_i^\theta(1-\tau)w_{sf} \geq w_{si}.$$

$$\eta^{**} = \left(\frac{w_{si}}{(1-\tau)w_{sf}} \right)^{\frac{1}{\theta}}. \tag{11}$$

Figure 1 shows this endogenous occupational choice. Low-skill workers whose efficiency units of labor are higher than η^* sort themselves into the goods to perform routine tasks, and those with efficiency units of labor lower than η^{**} sort themselves into the informal service sector.

Figure 1. Optimal Labor Choice

I assume that η is distributed Uniform on the interval $[0, \eta^{max}]$, with density and distribution functions $F(\eta)$ and $f(\eta)$ defined as follows:

$$F(\eta) = \begin{cases} \frac{\eta}{\eta^{max}} & 0 \le \eta \le \eta^{max} \\ 1 & \eta > \eta^{max} \end{cases}, \tag{12}$$

$$f(\eta) = \begin{cases} \frac{1}{\eta^{max}} & 0 \le \eta \le \eta^{max} \\ 0 & \eta < 0 \ or \ \eta > \eta^{max} \end{cases}.$$

The endogenous occupational choice of low-skill workers determines the effective labor supply in each sector. The aggregate efficiency units supplied to the routine tasks, manual tasks in the formal and informal service sector can be written, respectively, as follows:

9

$$L_r = \int_{\eta^*}^{\eta^{max}} \frac{\eta}{\eta^{max}} d\eta = \left[\frac{(\eta^{max})}{2} - \frac{(\eta^*)^2}{2\eta^{max}} \right], \tag{13}$$

$$L_{sf} = \int_{\eta^{**}}^{\eta^*} \frac{\eta^\theta}{\eta^{max}} d\eta = \left[\frac{(\eta^*)^{\theta+1}}{(\theta+1)\,\eta^{max}} - \frac{(\eta^{**})^{\theta+1}}{(\theta+1)\,\eta^{max}} \right], \tag{14}$$

$$L_{si} = \int_0^{\eta^{**}} \frac{1}{\eta^{max}} d\eta = \frac{\eta^{**}}{\eta^{max}}. \tag{15}$$

2.5 Households

I assume a representative household, whose members derive utility from the consumption of goods and formal and informal services. The household collects the wages of all its members and allocate total income to maximize the following utility function:

$$U = Ln\left(\left[\gamma_g C_g^\rho + \gamma_s C_s^\rho \right]^{\frac{1}{\rho}} \right), \tag{16}$$

with

$$C_s = \left(a_f C_{sf}^\psi + a_i C_{si}^\psi \right)^{\frac{1}{\psi}},$$

subject to the budget constraint.

$$C_g + P_{sf} C_{sf} + P_{si} C_{si} = (1-\tau)\left(w_a L^a + w_r L^r + w_{sf} L^{sf} \right) + w_{si} L^{si} + T. \tag{17}$$

The elasticity of substitution between C_g and C_s is equal to $\sigma_c = \frac{1}{1-\rho}$. The elasticity of substitution between C_{sf} and C_{si} is equal to $\sigma_s = \frac{1}{1-\psi}$. I assume that goods and services are complements, $\sigma_c < 1$, while formal and informal services are substitutes, $\sigma_s > 1$.

The First order conditions of this maximization problem are as follows:

$$\frac{C_g}{C_s} = \left(C_{sf}^{1-\psi} \frac{\gamma_g}{\psi a_f \gamma_s} \frac{P_{sf}}{P_g} \right)^{\sigma_c}, \tag{18}$$

$$\frac{C_{si}}{C_{sf}} = \left(\frac{a_i}{a_f} \frac{P_{sf}}{P_{si}} \right)^{\sigma_s}. \tag{19}$$

The left-hand side of the equation (18) represents the relative supply, while the right-hand side is the relative demand for goods compared to services. In the same way, equation (19) equals the relative supply of informal services, compared to formal services, with its relative demand.

2.6 Government

Government always runs a balanced budget. Therefore, in each period, government budget constraint is as follows:

$$T = \tau \left(w_a L_a + w_r L_r + w_{sf} L_{sf} \right). \tag{20}$$

2.7 Clearing conditions

$$C_g = Y^g - P_k K \tag{21}$$

$$C_{sf} = Y_{sf} \tag{22}$$

$$C_{si} = Y_{si} \tag{23}$$

3 Asymptotic Labor Allocation

In this section, I determine the log run allocation (asymptotic equilibrium) of low-skill workers in the goods, formal and informal service sectors. Given that the price of computer capital $P_k(t)$ converges to zero asymptotically, computer capital converges to infinity:

$$\lim_{t \to \infty} K(t) = \infty. \tag{24}$$

Since the maximum value of L_r is $\frac{(\eta^{max})}{2}$, the production of X will be asymptotically determined by the capital level ($X \sim \alpha_k K$). It implies that:

$$\lim_{t \to \infty} \frac{X}{\mu_k K} = 1, \tag{25}$$

and

$$Y_g \sim (\mu_k K)^\beta . \tag{26}$$

Replacing equation (26) into equation (21), and using equations (24) and (25), I show in Appendix C1, that the solution for the asymptotic supply of low-skilled labor in the goods and formal and informal service sector is as follows[4]:

---

[4]Here $\hat{L}_{si}$ is the solution to the equation $(\eta^{max})^{\theta+1} = (\theta+1)\eta^{max}\left(\frac{\hat{L}_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}\hat{L}_{si}\right)^{\theta+1}$. with $C_{11} = \left(\left(\frac{A_{si}}{A_{sf}}\right)^\psi \frac{a_i}{a_f(\eta^{max})^\theta(1-\tau)}\right)^{\frac{1}{\theta+1-\psi}}$. And $\bar{L}_{si}$ is the solution to the equation $\frac{gg(L_{si})^{1-\nu}(C_s(L_{si}))^{(\rho-1)}(L_{sf})^{\psi-1}}{\left((\theta+1)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max}L_{si})^{\theta+1}\right)^{-1}} = \Psi_1$,

where $\Psi_1 = \frac{\gamma_g \kappa_R A_g \mu_k^{\beta(\rho-1)+(\beta-\nu)}}{(1-\beta)^{1-\rho}\psi a_f \gamma_s A_{sf}^\psi}$.

11

$$L_r = \begin{cases} 0 & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \bar{L}_r \in \left(0, \frac{\eta^{max}}{2}\right) & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ \frac{\eta^{max}}{2} & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases}, \tag{27}$$

where $\bar{L}_r = gg(\bar{L}_{si})$.

$$L_{si} = \begin{cases} \hat{L}_{si} & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}, \\ \bar{L}_{si} \in \left(0, \hat{L}_{si}\right) & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases}, \tag{28}$$

$$L_{sf} = \begin{cases} \hat{L}_{sf} & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \bar{L}_{sf} \in \left(0, \hat{L}_{sf}\right) = \Psi & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases}, \tag{29}$$

where $\hat{L}_{sf} = \left(\frac{\hat{L}_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}$ and $\bar{L}_{sf} = \left(\frac{\bar{L}_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}$. $C_{11} = \left(\left(\frac{A_{si}}{A_{sf}}\right)^{\psi} \frac{a_i}{a_f(\eta^{max})^\theta(1-\tau)}\right)^{\frac{1}{\theta+1-\psi}}$

Equations (28), (29), and (27) show that, as in Autor and Dorn (2013), the allocation of low-skill labor between manual tasks and routine tasks depends on the relative magnitudes of the consumption and production elasticities, scaled by the share of the routine aggregate in goods production (β). When $\frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}$ (the production elasticity, scaled by β, exceeds the consumption elasticity) the demand for routine labor decreases with technological progress, and the relative demand for low-skill labor in both the formal and the informal service sector increases. Hence, the constant decrease in prices of automating routine tasks eventually causes all low-skill labor to flow from routine tasks to manual tasks. As a result, employment in the formal and informal service sectors increases.

The allocation of labor between the formal and the informal service sector is determined by the equation $L_{sf} = \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}$. It depends on the differences in productivity (A_{si} and A_{sf}), the level of labor income taxes paid by formal workers τ, consumer preferences (ψ, a_f, and a_i), and efficiency of labor in the formal sector $(\eta^{max})^\theta$. The higher the aggregate labor productivity and efficiency in the formal service sector, and the lower the labor income taxes, the higher the relative allocation of labor in the formal service sector.

Replacing equations (24) and (25) into equation (19), I have that the evolution of the ratio between the efficient units of labor in the informal and the formal sector depends exclusively on the evolution of the wage ratio between the two sectors.

$$\frac{L_{si}}{L_{sf}} = \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{a_i}{a_f} \frac{w_{sf}}{w_{si}}\right)^{\sigma_s}. \tag{30}$$

Additionally, from equation (11) the wage ratio $\frac{w_{si}}{w_{sf}}$ can be expressed as follows:

$$\frac{w_{si}}{w_{sf}} = (\eta^{**})^\theta (1 - \tau). \tag{31}$$

The wage ratio between the formal and the informal service sector depends on the level of labor income tax paid by formal workers and the efficiency level of the marginal worker, $(\eta^{**})^\theta$. The assumption that workers in the formal service sector are heterogeneous in their skill to perform manual tasks implies that this ratio is not constant and varies with the efficiency units of labor in the formal service sector. The higher the value of θ, the higher the worker's skills in the formal service sector (for those workers whose skill level is $\eta > 1$) compared with their skills in the informal service sector, and therefore the higher the relative informal wage. Note that for the case when all workers have the same ability to perform manual tasks activities in both service sectors (when $\theta = 0$), the wage ratio in the service sector $\frac{w_{si}}{w_{sf}}$ and hence the employment ratio $\frac{L_{si}}{L_{sf}}$ are constant and independent from technological progress.

Low-skill workers leaving the goods sector and entering the formal service sector are the ones that have relatively high efficiency, and those leaving the formal service sector are the ones that have relatively low efficiency. As a consequence, the average efficiency in the formal service sector increases.

4   Asymptotic wage inequality.

In this section, I study the evolution of wage inequality, measured by the evolution of manual to abstract, and manual to routine wage ratios, as well as the evolution of formal to informal wage ratios in the service sector. Using equations (4) and (18), and replacing the optimal conditions for wages in the service sector, (6) and (7), I show in Appendix C2 that the wage ratio $\frac{w_{sf}}{w_r}$ can be written as follows:

$$\frac{w_{sf}}{w_r} = \left((\theta + 1)\eta^{max} CC_1 L_{sf} + (\eta^{max} L_{si})^{\theta+1}\right)^{-1}.$$

Given the asymptotic labor allocation when $K \to \infty$

$$\frac{w_{sf}}{w_r} = \begin{cases} (\eta^{max})^{1-\theta} & if \quad \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta} \\ \left((\theta + 1)\eta^{max} CC_1 \bar{L}_{sf} + (\eta^{max} \bar{L}_{si})^{\theta+1}\right)^{-1} \in \left(0, (\eta^{max})^{1-\theta}\right) & if \quad \frac{1}{\sigma_c} = \frac{\beta - \nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta - \nu}{\beta} \end{cases} \tag{32}$$

On the other side, the wage ratio $\frac{w_{si}}{w_r}$ can be written as follows (see Appendix C2 for the complete derivation):

$$\frac{w_{si}}{w_r} = \left(\frac{L_{si}}{L_{sf}}\right)^{\psi-1} \Psi_1 \left((\theta+1)\eta^{max}L_{sf} + (\eta^{max}L_{si})^{\theta+1}\right)^{-1}.$$

Similarly, given the asymptotic labor allocation when $K \to \infty$ I obtain:

$$\frac{w_{si}}{w_r} = \begin{cases} \left((\theta+1)\eta^{max}\hat{L}_{sf} + \left(\eta^{max}\hat{L}_{si}\right)^{\theta+1}\right)^{-1} \left(\frac{\hat{L}_{si}}{\hat{L}_{sf}}\right)^{\psi-1} \Psi_1 & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \left((\theta+1)\eta^{max}\bar{L}_{sf} + \left(\eta^{max}\bar{L}_{si}\right)^{\theta+1}\right)^{-1} \left(\frac{\bar{L}_{si}}{\bar{L}_{sf}}\right)^{\psi-1} \Psi_1 & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \quad (33)$$

Equations (32) and (33) imply that the relative wage paid to manual tasks versus routine tasks increases with technological progress, for the case when the production elasticity (scaled by β) excess the consumption elasticity. Since Task-biased technological progress affects mainly the production of goods, and goods and services are complements, the aggregate demand for services also increases, consequently increasing the unit wage in service occupations relative to the unit wage of routine labor in the goods sector.

Additionally, from equations (31) and (15), the unit wage of manual labor in the informal service sector relative to the unit wage of manual labor in the formal service sector can be expressed as follows:

$$\frac{w_{si}}{w_{sf}} = \left(\eta^{max}L_{si}\right)^{\theta} (1-\tau). \qquad (34)$$

The evolution of the wage ratio $\frac{w_{si}}{w_{sf}}$ is thus determined by the evolution of the employment share in the informal service sector L_{si}. Scaled by the level of taxes (τ) and the efficiency levels in the formal service sector (θ). From equations (34) and (28), the asymptotic wage ratio $\frac{w_{si}}{w_{sf}}$ is determined as follows

$$\frac{w_{si}}{w_{sf}} = \begin{cases} \left(\eta^{max}\hat{L}_{si}\right)^{\theta}(1-\tau) & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \left(\eta^{max}\bar{L}_{si}\right)^{\theta}(1-\tau) & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \quad (35)$$

Given that $\hat{L}_{si} > \bar{L}_{si}$, equation (35) implies that the wage ratio between informal and formal occupations in the service sector increases with technological progress, for the case when $\frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}$. When the production elasticity (scaled by β) is bigger than the consumption elasticity, technological progress raises the relative demand for low-skill labor in the formal and the informal

service sectors, which requires a rise in wages in both sectors. Note that a worker endowed with η_i efficiency units of labor has individual labor productivity of η_i^θ when he works in the formal service sector, while the individual labor productivity of the same worker in the informal service sector is equal to 1. Since a worker with $\eta > 1$, is more efficient when he works in the formal service sector, the increase in wages in the informal service sector has to be higher than the one in the formal service sector in order to compensate for this loss of skills. The higher the worker's skill in the formal service sector (θ), the higher the increase in the wage ratio $\frac{w_{si}}{w_{sf}}$.

From equation (30), it is now possible to analyze the evolution of the employment composition in the service sector. This equation shows that when formal and informal services are substitutes (when $\frac{1}{1-\psi} > 1$), the efficiency units of labor employed in the informal sector relative to those employed in the formal service sector $\frac{L_{si}}{L_{sf}}$ decrease with the relative wage in the informal sector. The increase in $\frac{w_{si}}{w_{sf}}$ increases the relative price of informal services, thus lowering the relative demand for these services and therefore the relative demand for informal labor.

Finally, I determine the evolution of the wage ratio between wages in the service sector and wages in the abstract task. As Autor and Dorn (2013), wage polarization will occurs when $\frac{w_{sf}}{w_r}$ and $\frac{w_{sf}}{w_r}$ rises, and $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ are either stable or declining.

In Appendix C2, I show that the wage ratios $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{sI}}$ can be written as follows:

$$\frac{w_a}{w_{sf}} = \kappa_{af} K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)} C_s^{1-\rho} L_{sf}^{1-\psi},$$

$$\frac{w_a}{w_{si}} = \kappa_{ai} K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)} C_s^{1-\rho 1} L_{si}^{1-\psi},$$

with $\kappa_{af} = \frac{(1-\beta)(\mu_k)^\beta \gamma_g\left((1-\beta)(\mu_k)^\beta\right)^{\rho-1}}{(\gamma_s)a_f \psi A_{sf}^\psi}$, and $\kappa_{ai} = \kappa_{af}\left(\frac{A_{sf}}{A_{si}}\right)^\psi$.

As capital converge to infinity, the evolution of $\frac{w_a}{w_{sf}}$ is determined by the elasticity of substitution between goods and services, σ_c. Specifically,

$$\frac{w_a}{w_{sf}} = \begin{cases} \infty & if & \sigma_c > 1, \\ \kappa_{af}\hat{C}_s^{1-\rho}\hat{L}_{sf}^{1-\psi} & if & \sigma_c = 1 \quad with \; \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ 0 & if & \sigma_c < 1 \end{cases}, \tag{36}$$

$$\frac{w_a}{w_{sf}} = \begin{cases} \infty & if & \sigma_c > 1, \\ \kappa_{ai}\hat{C}_s^{1-\rho}\hat{L}_{si}^{1-\psi} & if & \sigma_c = 1 \quad with \; \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ 0 & if & \sigma_c < 1 \end{cases}. \tag{37}$$

Equations (36) and (37) show that when goods and services are gross complements $\sigma_c < 1$, the ratio between abstract tasks wages and manual tasks wages converge to zero. As a result, the model implies overall wage and job polarization due to a task-biased technological change.

5 Model simulations

In this section, I analyze the evolution of employment shares and sectoral wages as the price of capital converges to zero. I simulate the evolution of employment and wages under different values of the elasticity of substitution between formal and informal services σ_s. As equation (30) shows, the elasticity of the relative employment in the informal sector to the relative informal wages depends on σ_s. I also consider a different fiscal policy where labor income taxes are variable. The purpose of this exercise is to analyze how the job polarization process in developing and emerging countries, would affect the informal sector depending on preferences, and tax policies within each country.

All parameters are time-invariant, and the only exogenous change over time is the price of capital. I simulate the model for the case when the the production elasticity (scaled by β) excess the consumption elasticity (it means when $\frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}$). Under this scenario, independent from the values σ_c, β, and ν, technological progress always leads to job polarization. I choose standard values of the parameters commonly used in the literature. For the parameters describing preferences, I set the elasticity of substitution between goods and services at $\sigma_c = 0.4$, and the elasticity of substitution between formal and informal services at $\sigma_s = \frac{1}{0.7}$. These values imply goods and services are complements, and formal and informal services are substitutes. The share of routine aggregate in goods production is set at $\beta = 0.67$, as in Bock (2018). I normalize the aggregate labor productivity in the informal service to 1 and set aggregate labor productivity in the informal service sector equal to 0.71. It implies that the labor productivity in the formal sector is 39% higher than in the informal sector. This value is consistent with La Porta and Shleifer (2008) who find that the value-added per employee for registered firms is 39% higher than for their unregistered counterparts. I assumed similar weighs of routine labor and capital in the production of goods, and of goods and services in the utility function, this is $\mu_k = 0.5$, $\gamma_g = \gamma_s = 0.5$ and $a_f = a_i = 0.5$. The labor income tax is set at $\tau = 0.205$. This value is calibrated to obtain a level of tax revenue of 15% of GDP. According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 and 20 percent of GDP, while the average for high-income countries is more like 40 percent. Finally I set arbitrary values for δ_k, η^{max} and θ. Changes in these parameters do not affect the main result of the model. Table 1 summarizes the parameter values of the model.

Table 1. Parameter values: benchmark calibration

Name	Symbol	Value
Share of routine aggregate in goods production	β	0.67
Elasticity of substitution between consumption and services	σ_c	0.4
Elasticity of substitution between formal and informal services	σ_s	1/0.7
Inverse of the elasticity of substitution between routine labor and capital	ν	0.5
Parameter associated to the skill level of formal workers in the service sector	θ	0.7
Maximum skill level	η^{max}	2
Labor income tax	τ	0.205
Parameter reflecting technological progress	δ_k	0.01
Relative weigh of goods ans services in the utility function	$\gamma_g = \gamma_s$	0.5
Relative weigh of formal and informal services in the utility function	$a_f = a_i$	0.5
Relative weigh of capital in the production function of goods	μ_k	0.5
Aggregate labor productivity in the formal service sector	A_{sf}	1
Aggregate labor productivity in the informal service sector	A_{si}	0.71

Note that L_j represents the total amount of efficiency unit of labor employed in sector j, and w_j represents the wage per efficiency unit of labor in that sector, $j \in (r,\ si,\ sf)$. For the simulation, I also analyze the evolution of the employment shares and the average wages in each sector.

The low-skill employment share N_j is the mass of individuals who supply their labor in sector j, which are defined as follows:

$$N_r = \int_{\eta^*}^{\eta^{max}} f(\eta)d\eta = \left[1 - \frac{\eta^*}{\eta^{max}}\right], \tag{38}$$

$$N_{sf} = \int_{\eta^{**}}^{\eta^*} f(\eta)d\eta = \frac{1}{\eta^{max}}\left[\eta^* - \eta^{**}\right], \tag{39}$$

$$N_{si} = \int_0^{\eta^{**}} f(\eta)d\eta = \frac{\eta^{**}}{\eta^{max}}. \tag{40}$$

Additionally, the average wage in sector j is defined as the total labor income in sector j, divided by the mass of people working in this sector:

$$\bar{w}_j = \frac{w_j L_j}{N_j}, \qquad for \quad j \in (r, sf, si).$$

and in terms of the wage ratio between different sectors:

$$\frac{\bar{w}_j}{\bar{w}_i} = \frac{w_j}{w_i} \frac{L_j}{N_j} / \frac{L_i}{N_i}, \quad for \quad j \neq i.$$

Figure 2 shows the evolution of low-skill employment in the goods and the service sectors, under the benchmark calibration, when the price of capital converges to zero. In the goods sector, routine labor is substituted with capital, which decreases employment and the efficiency units of routine labor (N_r and L_r) in this sector. At the same time, the increase in the aggregate demand for services increases both raw employment (N_{si} and N_{sf}) and the efficiency units of labor in the formal and informal sectors (L_{si} and L_{sf}) . Consequently, the constant decrease in prices of automating routine tasks eventually causes all low-skill labor to flow from routine tasks to manual tasks. Moreover, it is worth noticing from Figure 2 that under this scenario, job polarization leads to a decrease in the ratio between the efficient units of labor in the informal and the formal sector L_{si}/L_{sf}. This decrease is due to the increase of relative informal unit wages in the service sector (see equation (30) and Figure 3). Additionally, by comparing the evolution of L_{si}/L_{sf} versus the relative employment share of the informal sector N_{si}/N_{sf}, it is possible to notice that the decrease on L_{si}/L_{sf} is higher than the decrease on N_{si}/N_{sf}, which implies that the formal service sector becomes more efficient.
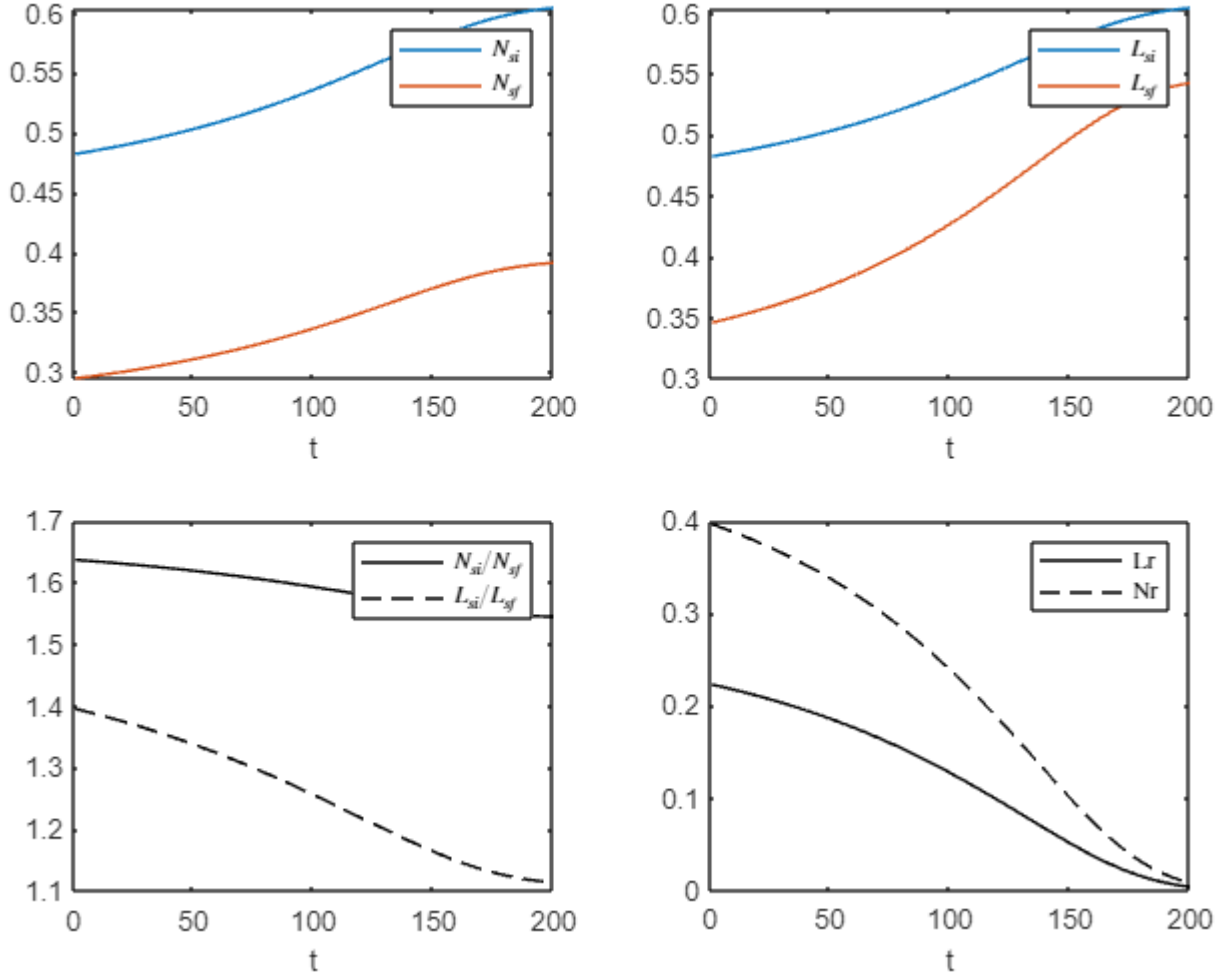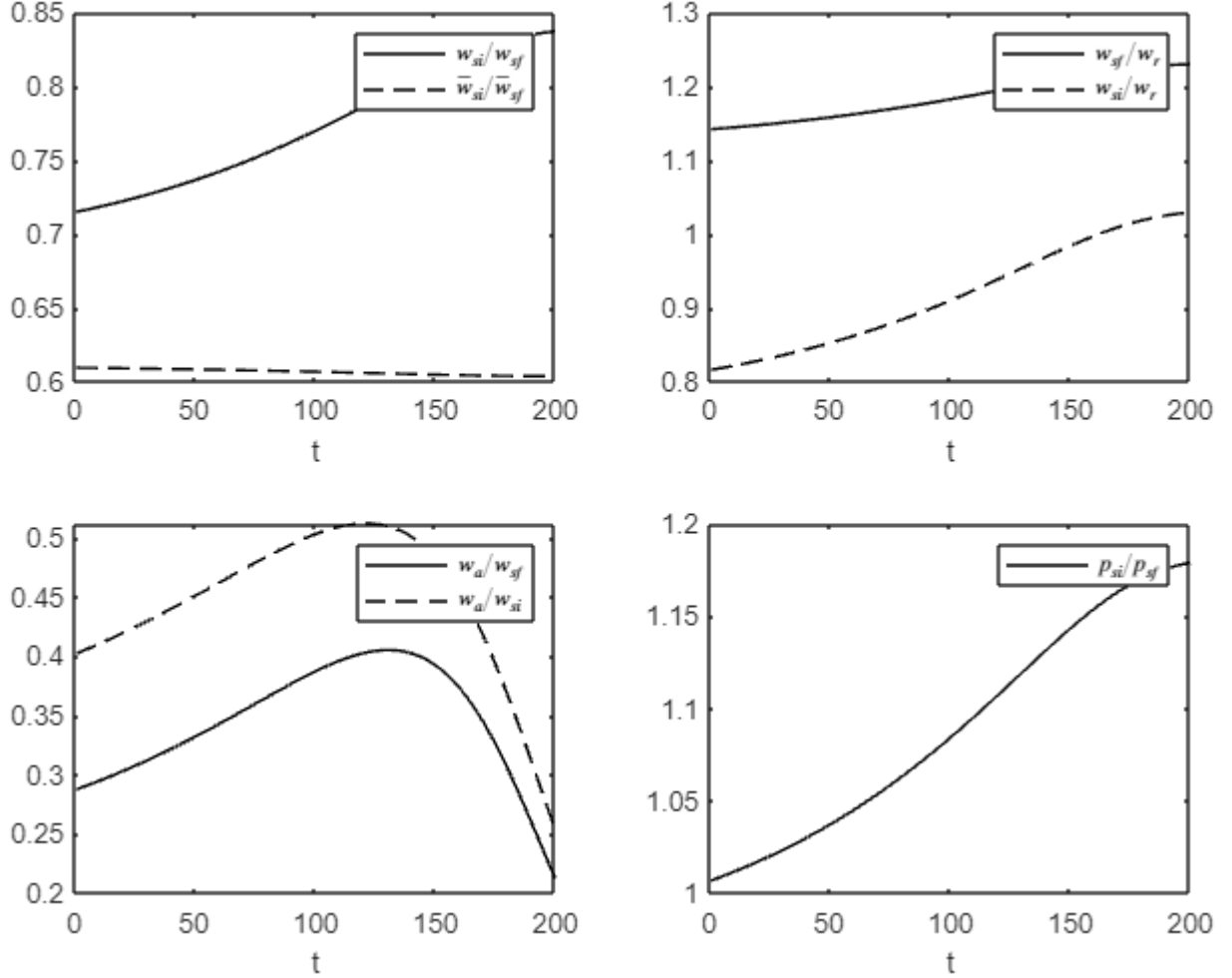
Figure 2. Evolution of employment

Figure 3 shows the evolution of relative prices and wages as the price of capital decreases. It shows that the ratio between informal and formal wages in the service sector (w_{si}/w_{sf}) increases. As was stated previously in section 4, technological progress raises the relative demand for low-skill labor in both service sectors, which requires a rise in wages in each sector. Workers with ability $\eta > 1$ are more skilled when they work in the formal sector. Therefore, the increase in wages in the informal service sector has to be higher than the increase in the formal service sector to compensate for this loss of skills. As a result, the increase in the relative informal wage increases the relative price of informal services and, as a consequence, it reduces their relative demand.

In terms of average wages, it is worth noticing that the relative average wage in the informal service sector versus the average wage in the formal sector, $\bar{w}_{si}/\bar{w}_{sf}$, is almost constant when the price of capital decreases. It implies that, on average, the wage differences between formal and informal workers remain constant with technological progress. Notice also that the wage ratio between manual tasks in the service sector and routine tasks increases, while the wage

19

ratio between abstract tasks and manual tasks increases initially and then converges to zero. This result shows that when $\sigma_c < 1$, wages polarize in the long run(see equation (36)).

Figure 3 Evolution of relative wages and prices in the service sector

In order to better understand the effect of technological progress and tax policy on the composition of employment between sectors, I analyze the evolution of the share of low skill employment in the goods and the service sector when the price of capital decreases ($\downarrow P_k$) and labor income taxes in the formal sector increases ($\uparrow \tau$). Figure 4 shows that a decrease in the price of capital decreases the share of routine labor (N_r) in the goods sector, almost independent from the level of taxes. The decrease in taxes however affects the distribution of workers in the service sector. For a given level of taxes, the decrease in the price of capital increases employment in both the formal and the informal service sectors, N_{sf} and N_{si} respectively. An increase in the level of taxes increases the share of employment in the informal service sector and decreases the share of employment in the formal service sector. These results suggest that technological progress combined with an important decrease in taxes could lead to a decrease in the size of the informal sector.

**Figure 4. Effect of a decrease on the price of capital and an increase
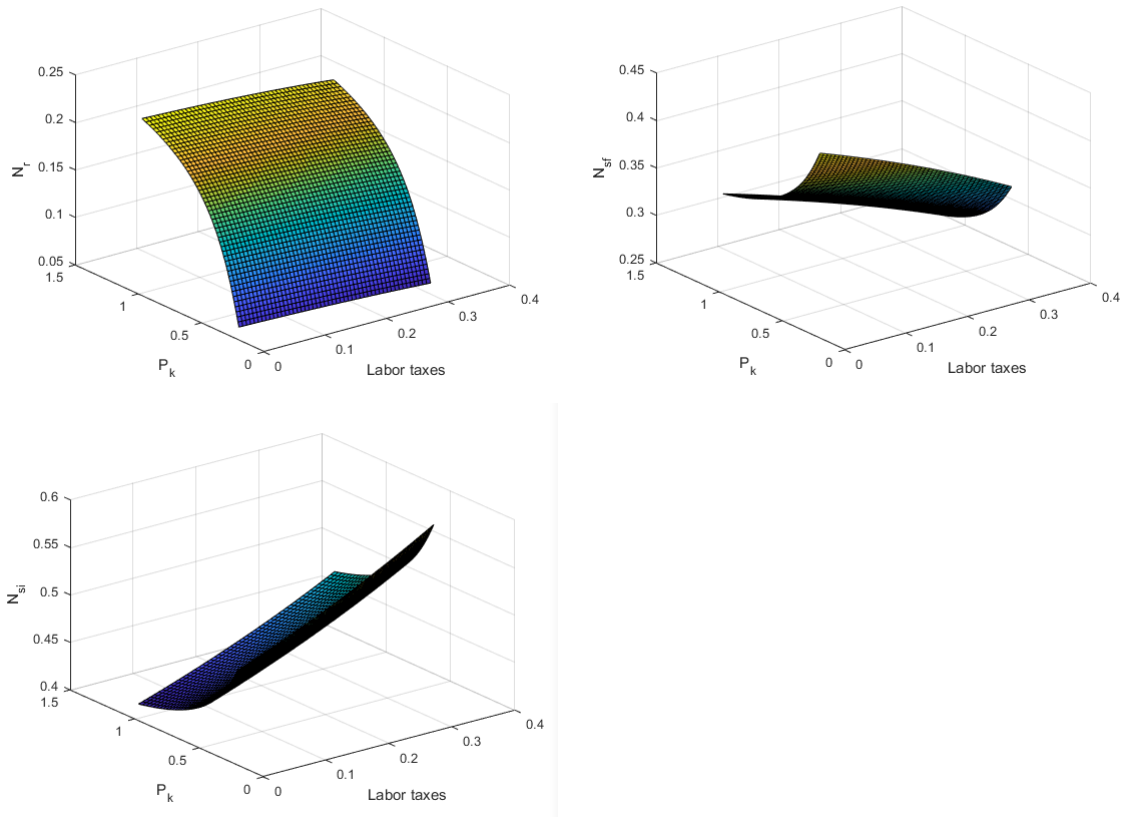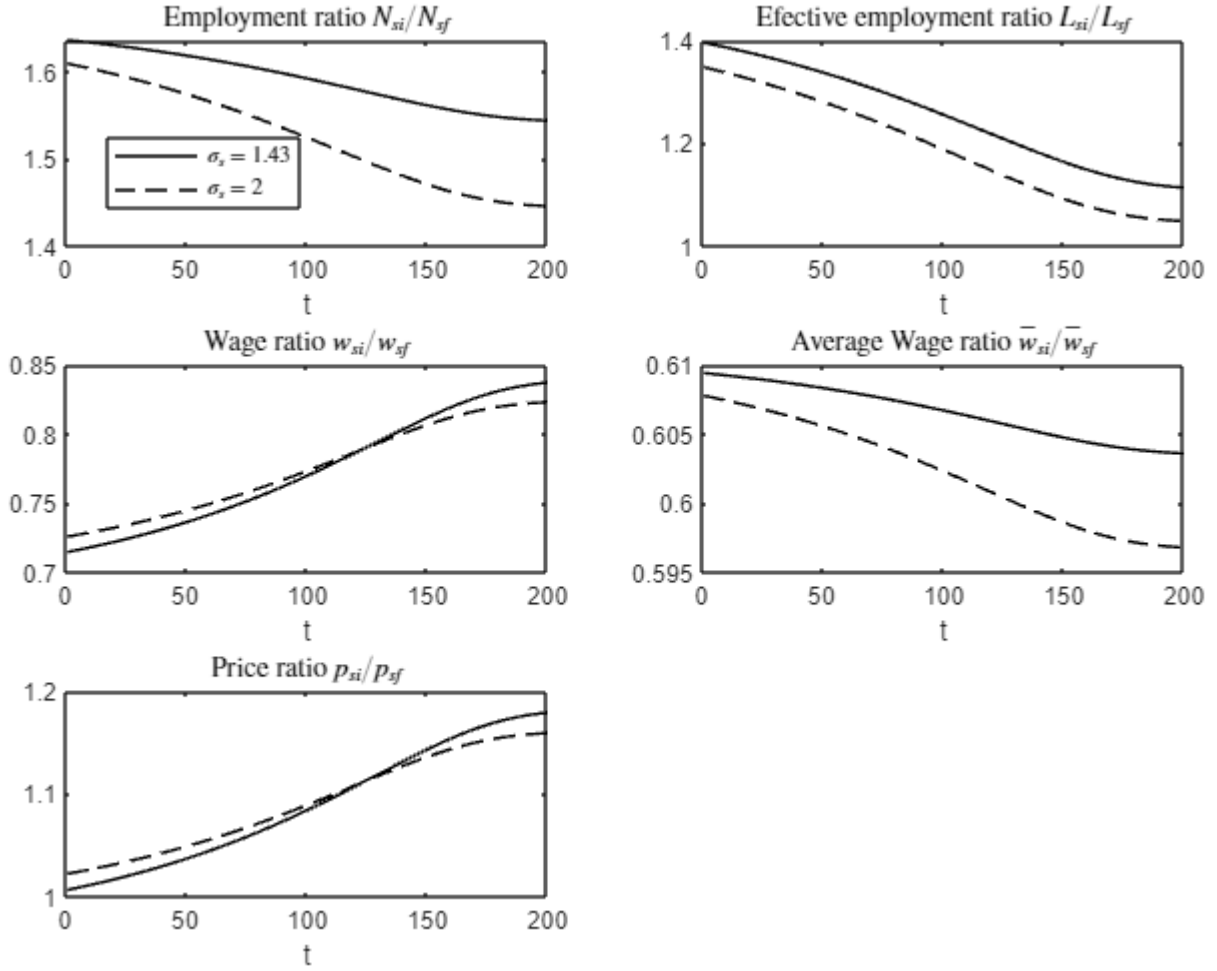on labor income taxes on employment**

Figure 5 shows the evolution of relative employment, prices, and wages in the service sector when the price of capital decreases and the degree of substitution between formal and informal services is higher. When σ_s is higher (increases from 1.43 to 2), the elasticity of the relative informal employment with respect to relative informal wages increases. Therefore, the decrease in the relative demand of informal labor in the service sector is higher, since consumer's demand for formal services is higher.

Figure 5. Evolution of employment, prices and wages in the service sector for different values of σ_s

Employment ratio N_{si}/N_{sf}

$\sigma_s = 1.43$
$\sigma_s = 2$

Efective employment ratio L_{si}/L_{sf}

Wage ratio w_{si}/w_{sf}

Average Wage ratio $\bar{w}_{si}/\bar{w}_{sf}$

Price ratio p_{si}/p_{sf}

6 Conclusions

This paper analyzes how the incidence of job polarization affects the distribution of employment and wages in the presence of a large informal sector. I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorn (2013). I assume that there are three sectors in this economy: the goods sector, the formal service sector, and the informal service sector. Workers in the informal service sector are at the bottom of the skill distribution, are less productive, and can avoid taxation.

The analytical solution of the model implies that when the elasticity of substitution between capital and routine labor is higher than the elasticity of substitution between goods and services, the constant decrease in the price of capital eventually causes low-skill labor flows from routine tasks to manual tasks. This condition for job polarization is the same found by Autor and Dorn (2013). In this case, Task-biased technological progress can increase aggregate demand

for services and eventually increase employment and wages in service occupations. Additionally, when goods and services are complements, wages also polarize.

Additionally, I find that the efficient units of labor, as well as the number of workers, hired in the informal sector increase as a result of the increasing demand for informal services. However, the optimal composition of employment in the service sector depends on the level of taxes in the formal service sector τ, the degree of substitution between the two types of services σ_s, and the level of efficiency in the formal service sector η^θ. I find that share of informal employment in the service sector decreases with technological progress. This result is explained by the fact that some workers, whose skill level is $\eta > 1$, are more skilled when they work in the formal sector. Therefore, the increase in the demand for labor in the informal service sector requires a higher increase in wages to compensate for this loss of skills. As a consequence, the increase in relative wages in the informal sector increases their relative prices, which in turn decreases their relative demand and hence the relative demand for workers in the informal service sector.

I simulate the model for different values of the elasticity of substitution between formal and informal services and also for the case when labor income taxes are variable. I find that, when the elasticity of substitution between formal and informal goods increases, the effect of technological progress on the reduction of the share of informal employment in the service sector is higher. This is because consumers are more likely to substitute informal services when their relative price increases. I also analyze the evolution of employment in the goods and the service sector for different values of the price of capital and labor income taxes. I find that technological progress (decrease in the price of capital) combined with a significant decrease in taxes could lead to a reduction in the size of the informal sector.

The previous results are mainly driven by the assumption of flexible labor markets in all sectors and by the fact that some workers are more skilled when they work in the formal sector. One interesting extension of the model will be the introduction of wage and employment rigidities in the formal sector, while the informal sector is frictionless. Under this scenario, the positive effect of job polarization outlined in this paper can be diminished or even reversed. When task-biased technological progress leads to an increase in labor demand in the service sector, the presence of real wage rigidities and search and matching frictions in the formal sector, could lead to a higher increase in employment in the informal service sector. Since wages in the formal sector could not increase that much, and also there are some restrictions to enter into the formal sector. Another interesting extension would be to consider the fact that workers can receive education and acquire specific skills. Under this scenario, technological progress could lead to a decrease in the share of informal employment in the whole economy. Because more educated workers would either work in the goods sector performing abstract tasks or stay in the formal service sector where workers are remunerated according to their skills, the size of the informal sector will decrease.

Appendix A1: Asymptotic labor allocation

From the optimization conditions for the households I have

$$\max_{\{C_g, C_{sf}, C_{si}\}} U = Ln\left(C\right)$$

$$U = Ln\left(C\right), \tag{41}$$

with

$$C = \left[\gamma_g C_g^\rho + \gamma_s C_s^\rho\right]^{\frac{1}{\rho}}, \tag{42}$$

$$C_s = \left(a_f C_{sf}^\psi + a_i C_{si}^\psi\right)^{\frac{1}{\psi}}$$

$$P_g C_g + P_{sf} C_{sf} + P_{si} C_{si} = (1 - \tau)\left(w_a L^a + w_r L^r + w_{sf} L_{sf}\right) + w_{si} L_{si} + T$$

The Lagrangian of this problem writes:

$$\mathcal{L} = ln\left[\gamma_g C_g^\rho + \gamma_s \left(a C_{sf}^\psi + (a_i) C_{si}^\psi\right)^{\frac{\rho}{\psi}}\right]^{\frac{1}{\rho}}$$
$$+ \lambda\left((1 - \tau)\left(w_a L_a + w_r L_r + w_{sf} L_{sf}\right) + w_{si} L^{si} + T - P_g C_g - P_{sf} C_{sf} - P_{si} C_{si}\right)$$

F.O.C

$$\{C_g\} \qquad [...]^{\frac{1}{\rho} - 1} \gamma_g C_g^{\rho - 1} = P_g \lambda \tag{43}$$

$$\{C_{sf}\} \qquad [...]^{\frac{1}{\rho} - 1} \gamma_s C_s^{\rho - 1} a_f \psi C_{sf}^{\psi - 1} = \lambda P_{sf} \tag{44}$$

$$\{C_{si}\} \qquad [...]^{\frac{1}{\rho} - 1} \gamma_s C_s^{\rho - 1} (a_i) \psi C_{si}^{\psi - 1} = \lambda P_{si} \tag{45}$$

By dividing equations (43) and (44), and equations (45) and (44)I obtain:

$$\frac{C_g}{C_s} = \left(C_{sf}^{1 - \psi} \frac{\gamma_g}{\psi a_f \gamma_s} \frac{P_{sf}}{P_g}\right)^{\sigma_c} \tag{46}$$

$$\frac{C_{si}}{C_{sf}} = \left(\frac{a_i}{a_f} \frac{P_{sf}}{P_{si}}\right)^{\sigma_s} \tag{47}$$

Replacing $C_{si} = A_{si} L_{si}$ and $C_{sf} = A_{sf} L_{sf}$ into equation (47) I obtain

$$L_{si} = L_{sf} \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{a_i}{a_f} \frac{w_{sf}}{w_{si}}\right)^{\frac{1}{1-\psi}}$$

from equation (11) I have $\frac{w_{sf}}{w_{si}} = \frac{1}{(\eta^{**})^\theta (1-\tau)}$. Replacing this expression into the previous equation:

$$L_{si} = L_{sf} \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{(a_i)}{a_f} \frac{1}{(\eta^{**})^\theta (1-\tau)}\right)^{\frac{1}{1-\psi}}$$

Using equation (15) I can express $\eta^{**} = \eta^{max} L_{si}$, then

$$L_{si} = L_{sf} \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{(a_i)}{a_f} \frac{1}{(\eta^{max} L_{si})^\theta (1-\tau)}\right)^{\frac{1}{1-\psi}}$$

$$L_{si}^{\frac{\theta+1-\psi}{1-\psi}} = L_{sf} \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{a_i}{a_f (\eta^{max})^\theta (1-\tau)}\right)^{\frac{1}{1-\psi}}$$

$$L_{si} = L_{sf}^{\frac{1-\psi}{\theta+1-\psi}} C_{11} \tag{48}$$

where $C_{11} = \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{\theta+1-\psi}} \left(\frac{a_i}{a_f (\eta^{max})^\theta (1-\tau)}\right)^{\frac{1}{\theta+1-\psi}}$

On the other side, replacing equations (4), (21), and (26) into equation (46) I have:

$$\frac{C_g}{C_s} = \left(C_{sf}^{1-\psi} \frac{\gamma_g}{\psi a_f \gamma_s} \frac{P_{sf}}{P_g}\right)^{\sigma_c}$$

$$\gamma_g (Y^g - P_k K)^{\frac{-1}{\sigma_c}} \frac{P_{sf}}{P_g} = \gamma_s C_s^{\frac{-1}{\sigma_c}} \psi a_f C_{sf}^{\psi-1}$$

$$\gamma_g (Y^g - P_k K)^{\rho-1} \frac{\kappa_R A_g L_a^{(1-\beta)} X^{\beta-\nu} L_r^{\nu-1}}{A_{sf}} \frac{w_{sf}}{w_r} = \gamma_s (C_s(L_{si}))^{(\rho-1)} \psi a_f (A_{sf} L_{sf})^{\psi-1}$$

From (10) I have that $\frac{w_{sf}}{w_r} = (\eta^*)^{1-\theta}$. replacing this equation into the previous equation I obtain:

$$\gamma_g (Y^g - P_k K)^{\rho-1} \frac{\kappa_R A_g L_a^{(1-\beta)} X^{\beta-\nu} L_r^{\nu-1}}{A_{sf}} (\eta^*)^{1-\theta} = \gamma_s (C_s(L_{si}))^{(\rho-1)} \psi a_f (A_{sf} L_{sf})^{\psi-1}$$

with $Y_g - P_k K = (1-\beta)(\mu_k K)^\beta$ and $\frac{X_t}{\mu_k K} = 1$, $L_{si} = L_{sf}^{\frac{1-\psi}{\theta+1-\psi}} C_{11}$

$$\gamma_g \left((1-\beta)(\mu_k K)^\beta\right)^{\rho-1} \frac{\kappa_R A_g L_a^{(1-\beta)} (\mu_k K)^{\beta-\nu} L_r^{\nu-1}}{A_{sf}} (\eta^*)^{1-\theta} = \gamma_s (C_s(L_{si}))^{(\rho-1)} \psi a_f (A_{sf} L_{sf})^{\psi-1}$$

$$\frac{L_r^{1-\nu}}{(\eta^*)^{1-\theta}} (C_s(L_{si}))^{(\rho-1)} L_{sf}^{\psi-1} = K^{-(\beta)(1-\rho)+(\beta-\nu)} \Psi_1 \tag{49}$$

25

with $\Psi_1 = \frac{\gamma_g \kappa_R A_g \mu_k^{\beta(\rho-1)+(\beta-\nu)}}{(1-\beta)^{1-\rho}\psi a_f \gamma_s A_{sf}^{\psi}}$.

Now I want to express equation (49) in terms of L_{si} only.

Combining equations (39) , (40) and (48) I obtain:

$$\eta^* = \left((\theta+1)\,\eta^{max} \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max}L_{si})^{\theta+1} \right)^{\frac{1}{\theta+1}} \tag{50}$$

Replacing (50) into (13) I am able to express L_r as a function of L_{si}:

$$L_r = \left[\frac{(\eta^{max})}{2} - \frac{\left(\left((\theta+1)\,\eta^{max} \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max}L_{si})^{\theta+1} \right)^{\frac{1}{\theta+1}} \right)^2}{2\eta^{max}} \right] = gg(L_{si}) \tag{51}$$

Therefore, equation (49) can be written as follows:

$$\frac{gg(L_{si})^{1-\nu}\,(C_s(L_{si}))^{(\rho-1)} \left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} \right)^{\psi-1}}{\left((\theta+1)\,\eta^{max} \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max}L_{si})^{\theta+1} \right)^{\frac{1-\theta}{\theta+1}}} = K^{-(\beta)(1-\rho)+(\beta-\nu)}\Psi_1$$

with $\sigma_c = \frac{1}{1-\rho}$

$$\frac{gg(L_{si})^{1-\nu}\,(C_s(L_{si}))^{(\rho-1)} \left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} \right)^{\psi-1}}{\left((\theta+1)\,\eta^{max} \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max}L_{si})^{\theta+1} \right)^{\frac{1-\theta}{\theta+1}}} = K^{-\frac{(\beta)}{\sigma_c}+(\beta-\nu)}\Psi_1 \tag{52}$$

Appendix A2: Asymptotic wage inequality

From equations (43) and (44) I find

$$[...]^{\frac{1}{\rho}-1}(\gamma_s)\,C_s^{\rho-1}a\psi C_{sf}^{\psi-1} = [...]^{\frac{1}{\rho}-1}\gamma_g C_g^{\rho-1}P_{sf}$$

$$(\gamma_s)\,C_s^{\rho-1}a\psi C_{sf}^{\psi-1} = \gamma_g \left((1-\beta)(\mu_k K)^{\beta} \right)^{\rho-1}\frac{w_{sf}}{A_{sf}}$$

$$w_{sf} = \frac{\gamma_s C_s^{\rho-1}a\psi C_{sf}^{\psi-1}A_{sf}}{\gamma_g \left((1-\beta)(\mu_k K)^{\beta} \right)^{\rho-1}} \tag{53}$$

From the optimization problem for the firm in the goods and service sector, I have

$$w_r = \kappa_R \left(\mu_k K\right)^{\beta-\nu} g(L_{sf})^{\nu-1} \tag{54}$$

$$w_a = P_g \left(1-\beta\right)\frac{Y_g}{L_a} = \left(1-\beta\right)\left(\mu_k K\right)^{\beta} \tag{55}$$

$$w_{sf} = P_{sf}A_{sf}$$

$$w_{si} = P_{si}A_{si}$$

from equations (53) and (54) I obtain:

$$\frac{w_{sf}}{w_r} = \frac{C_s^{\rho-1}\left(L_{sf}\right)^{\psi-1}\left(g(L_{sf})\right)^{1-\nu}}{\kappa_{sf}\left(K\right)^{-\beta(1-\rho)+\beta-\nu}}$$

$$\frac{w_{sf}}{w_r} = \frac{C_s^{\rho-1}\left(L_{sf}\right)^{\psi-1}\left(g(L_{sf})\right)^{1-\nu}}{\kappa_{sf}\left(K\right)^{-\beta(1-\rho)+\beta-\nu}} \tag{56}$$

with $\kappa_{sf} = \frac{\alpha^{\rho-1}\kappa_R(\mu_k)^{(\beta)(\rho-1)+\beta-\nu}}{(A_{sf})^{\psi-1}\left(\psi\alpha\frac{\gamma_s}{\gamma_g}\right)^2}$

Replacing the expression $L_{si} = L_{sf}^{\frac{1-\psi}{\theta+1-\psi}}C_{11}$

$$\frac{w_{sf}}{w_r} = \frac{C_s^{\rho-1}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{\psi-1}\left(gg(L_{si})\right)^{1-\nu}}{\kappa_{sf}\left(K\right)^{-\beta(1-\rho)+\beta-\nu}}$$

Replacing equation (52) I have

$$\frac{w_r}{w_{sf}} = \frac{\left((\theta+1)\eta^{max}CC_1\left(\bar{L}_{si}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}\bar{L}_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}\left(K_t\right)^{(\beta)(\rho-1)+\beta-\nu}\Psi_1}{\kappa_{sf}K^{-(\beta)(1-\rho)+(\beta-\nu)}}$$

$$\frac{w_r}{w_{sf}} = \left((\theta+1)\eta^{max}CC_1\left(\bar{L}_{si}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}\bar{L}_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}\frac{\Psi_1}{\kappa_{sf}}$$

Similarly, the wage ratio between informal manual tasks and routine tasks $\frac{w_{si}}{w_r}$ can be determined as follows:

By dividing equation (43) into (45) I obtain:

$$\frac{\gamma C_g^{\rho-1}}{\gamma_s C_s^{\rho-1}\psi\left(\beta\right)C_{si}^{\psi-1}} = \frac{P_g}{P_{si}}$$

Replacing $w_{si} = P_{si}A_{si}$, and $C_g = Y^g - P_k K = \alpha\left(\mu_k K\right)^{\beta}$ I have

$$\frac{\left((1-\beta)\left(\mu_k K\right)^\beta\right)^{\rho-1}}{[C_s]^{\rho-1}\psi\beta\left(A_{si}L_{si}\right)^{\psi-1}} = \frac{\gamma_s A_{si}}{\gamma_g w_{si}}$$

$$w_{si} = C_s^{\rho-1}\psi\frac{\gamma_s\beta}{\gamma_g}A_{si}^\psi L_{si}^{\psi-1}\left((1-\beta)\left(\mu_k K\right)^\beta\right)^{1-\rho} \tag{57}$$

Dividing equation (57) into (54), I obtain

$$\frac{w_{si}}{w_r} = \frac{C_s^{\rho-1}\psi\frac{\gamma_s\beta}{\gamma_g}A_{si}^\psi L_{si}^{\psi-1}}{\left((1-\beta)\left(\mu_k K\right)^\beta\right)^{\rho-1}\kappa_R\left(\mu_k K\right)^{\beta-\nu}g(L_{sf})^{\nu-1}}$$

$$\frac{w_{si}}{w_r} = \frac{\kappa_{si}C_s^{\rho-1}L_{si}^{\psi-1}gg(L_{si})^{\nu-1}}{K^{-\beta(1-\rho)+(\beta-\nu)}}$$

with $\kappa_{si} = \frac{\psi\beta\gamma_s A_{si}^\psi\left((1-\beta)(\mu_k)^\beta\right)^{1-\rho}}{\gamma_g\kappa_R(\mu_k)^{\beta-\nu}}$

Replacing equation (52) I have

$$\frac{gg(L_{si})^{1-\nu}(C_s)^{(\rho-1)}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{\psi-1}}{\left((\theta+1)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+(\eta^{max}L_{si})^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}} = K^{-\frac{(\beta)}{\sigma_c}+(\beta-\nu)}\Psi_1$$

$$\frac{w_{si}}{w_r} = \frac{\kappa_{si}C_s^{\rho-1}L_{si}^{\psi-1}gg(L_{si})^{1-\nu}}{K^{-\beta(1-\rho)+(\beta-\nu)}}$$

$$\frac{w_{si}}{w_r} = \frac{\left((\theta+1)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+(\eta^{max}L_{si})^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}K^{-\frac{(\beta)}{\sigma_c}+(\beta-\nu)}\Psi_1 L_{si}^{\psi-1}}{K^{-\beta(1-\rho)+(\beta-\nu)}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{\psi-1}}$$

$$\frac{w_{si}}{w_r} = \left((\theta+1)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+(\eta^{max}L_{si})^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{1-\psi}\kappa_{si}\Psi_1 L_{si}^{\psi-1}$$

$$\frac{w_{si}}{w_r} = \left((\theta+1)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+(\eta^{max}L_{si})^{\theta+1}\right)^{-1}\left(\frac{L_{sf}}{L_{si}}\right)^{1-\psi}\kappa_{si}\Psi_1$$

or

$$\frac{w_{si}}{w_r} = \left((\theta+1)\eta^{max}L_{sf}+(\eta^{max}L_{si})^{\theta+1}\right)^{-1}\left(\frac{L_{sf}}{L_{si}}\right)^{1-\psi}\kappa_{si}\Psi_1$$

Finally, by dividing equation (55) into (53), and equation (55) into (57) I obtain the wage ratios $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ respectively:

$$\frac{w_a}{w_{sf}} = \frac{(1-\beta)(\mu_k)^\beta \gamma_g \left((1-\beta)(\mu_k)^\beta\right)^{\rho-1} K^{\beta\left(1-\frac{1}{\sigma_c}\right)}}{(\gamma_s) C_s^{\rho-1} a\psi (A_{sf} L_{sf})^{\psi-1} A_{sf}}$$

$$\frac{w_a}{w_{sf}} = \kappa_{af} K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)} C_s^{1-\rho} L_{sf}^{1-\psi}$$

where $\kappa_{af} = \frac{(1-\beta)(\mu_k)^\beta \gamma_g \left((1-\beta)(\mu_k)^\beta\right)^{\rho-1}}{(\gamma_s) a_f \psi A_{sf}^\psi}$

$$\frac{w_a}{w_{si}} = \frac{(1-\beta)(\mu_k K)^\beta}{C_s^{\rho-1} \psi \frac{\gamma_s \beta}{\gamma_g} A_{si}^\psi L_{si}^{\psi-1} \left(\alpha (\mu_k K)^\beta\right)^{1-\rho}}$$

$$\frac{w_a}{w_{si}} = \frac{(1-\beta)\mu_k^\beta K^{\beta\left(1-\frac{1}{\sigma_c}\right)}}{C_s^{\rho-1} \psi \frac{\gamma_s \beta}{\gamma_g} A_{si}^\psi L_{si}^{\psi-1} \left((1-\beta)\mu_k^\beta\right)^{1-\rho}}$$

$$\frac{w_a}{w_{si}} = \frac{\kappa_{ai} K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)}}{C_s^{\rho-1} L_{si}^{\psi-1}}$$

where $\kappa_{ai} = \frac{(1-\beta)(\mu_k)^\beta \gamma_g \left((1-\beta)(\mu_k)^\beta\right)^{\rho-1}}{(\gamma_s) a_f \psi A_{si}^\psi}$

References

Acemoglu, D, & Autor, D. (2011). Skills, Tasks and Technologies: Implications for Employment and Earnings. Handbook of Labor Economics, in: O. Ashenfelter & D. Card (ed.), Handbook of Labor Economics, edition 1, 4(12), 1043-1171.

Autor, D. H., Levy, F., & Murnane, R. J. (2003). The Skill Content of Recent Technological Change: An Empirical Exploration. *Quarterly Journal of Economics,* 118 (4), 1279– 1333.

Autor, D H., Dorn, D., & Gordon, H. (2015). Untangling trade and technology: evidence from local labour markets. *The Economic Journal,* 125, 621–646.

Autor, D H., Katz, L. F., & Kearney, M. S. (2006). The Polarization of the U.S. Labor Market. *American Economic Review,* 96 (2), 189–94.

Autor, D. H., & Dorn, D. (2013). The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market. *American Economic Review,* 103 (5), 1553–97.

Bachas, P. Kondylis, F., & Loeser, J. (2021). Increasing tax revenue in developing countries. Development Impact, World Bank Blog.

Besley, T., & Persson, T. (2014). Why Do Developing Countries Tax So Little? *Journal of Economic Perspectives*, 28(4), 99–120.

Blades, D. Ferreira, F., & Lugo, M.A. (2011). The informal economy in developing countries: an Introduction. *Review of Income and Wealth*, Series 57, Special Issue, May.

Blinder, Alan S. (2009). How Many US Jobs Might be Offshorable? World Economics, London, United Kingdom, 10(2), 41-78.

Bock, Sébastien. (2018). Job Polarization and Unskilled Employment Losses in France. PSE Working Papers halshs-01513037, HAL.

Docquier, F., Müller, T., & Naval, J. (2017). Informality and Long-Run Growth. *The Scandinavian Journal of Economics,* 119(4), 1040-1085

Elgin, C., & Birinci, S. (2016). Growth and informality: A comprehensive panel data analysis. J*ournal of Applied Economics*, 19, 271-292.

Feng, A., & Graetz, G. (2015). Rise of the Machines: The Effects of Labor-Saving Innovations on Jobs and Wages. Institute for the Study of Labor (IZA) Discussion Paper 8836.

Goos, M., Manning, A., & Salomons. A. (2014). Explaining Job Polarization: Routine-Biased Technological Change and Offshoring. *American Economic Review,* 104 (8), 2509–26.

ILO(2018). Women and men in the informal economy: a statistical picture (third edition). International Labour Office – Geneva: ILO, 2018.

ILO (2018b). New technologies and the transition to formality: The trend towards e–formality. International Labor Ofice, Geneva ISSN 1999-2939; 1999-2947.

Keller, W., & Utar, H. (2016). International Trade and Job Polarization: Evidence at the Worker-Level. CESifo Working Paper Series 5978.

La Porta, R., & Shleifer, A. (2014). Informality and Development. *Journal of Economic Perspectives*, 28(3), 109–126.

La Porta, R., & Shleifer, A. (2008). The Unofficial Economy and Economic Development. Brookings Papers on Economic Activity. 275-352.

Loayza, Norman (1996). The Economics of the Informal Sector: A Simple Model and Some Empirical Evidence from Latin America. Carnegie-Rochester Conference Series on Public Policy, 45(1), 129-62.

Loayza, N., Oviedo, A. M., & Servén, L. (2005). Regulation and Firm Dynamics. mimeo, The World Bank .

Michaels, G., Natraj, A., & Van Reenen, J. (2014). Has ICT Polarized Skill Demand? Evidence from Eleven Countries over Twenty-Five Years. *Review of Economics and Statistics,* 96 (1), 60–77.

Reijnders, L., & de Vries, G. J. (2018). Technology, offshoring and the rise of non-routine jobs. *Journal of Development Economics*, 135(C), 412-432.

World Bank (2016). World Development Report 2016: Digital Dividends. Washington, DC: World Bank. doi:10.1596/978-1-4648-0671-1. License: Creative Commons Attribution CC BY 3.0 IGO

Wu, D. F., & Schneider, F. (2019). Nonlinearity Between the Shadow Economy and Level of Development. IMF Working Papers 2019/048, International Monetary Fund.